Quiz 3 – 11.6

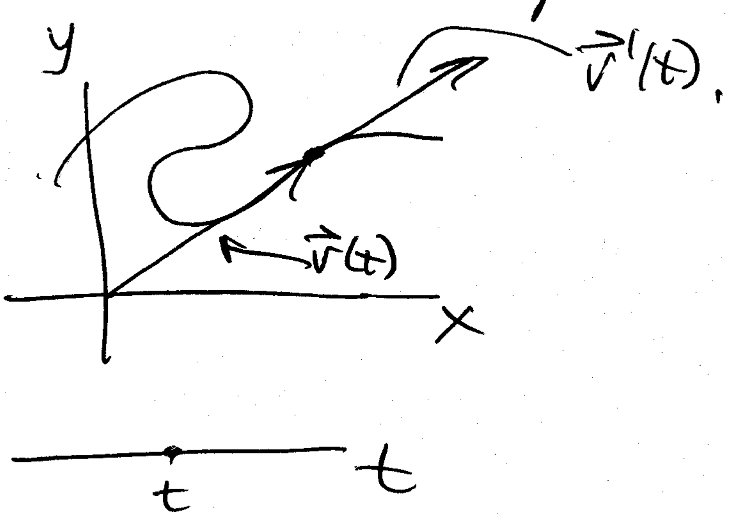
Exam 1 – Tuesday 2-21 – Cover 11.1–11.9.

$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

Graph of $\vec{r}(t)$ is a curve parametrized by $(x(t), y(t), z(t))$.

Think of $\vec{r}(t)$ as the position function of some object at time t.

$\vec{r}'(t) = x'(t)\vec{i} + y'(t)\vec{j} + z'(t)\vec{k}$ (velocity) is a vector tangent to curve.

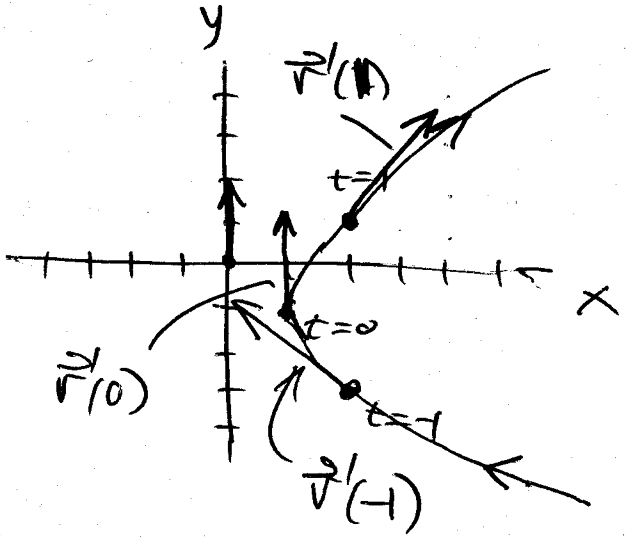


$|\vec{r}'(t)|$ is interpreted as speed, i.e. rate of change of displacement.

e.g. $\vec{r}(t) = (t^2+1)\vec{i} + (2t-1)\vec{j}$

Find path, velocity, speed of particle.

Path:



$\vec{r}(0) = \vec{\imath} - \vec{\jmath}$

$\vec{r}(1) = 2\vec{\imath} + \vec{\jmath}$

$\vec{r}(-1) = 2\vec{\imath} - 3\vec{\jmath}$

Find path precisely

Eliminate parameter:

$x = t^2 + 1 \rightarrow x - 1 = t^2$

$y = 2t - 1 \rightarrow t = \frac{y+1}{2}$

$x - 1 = \left(\frac{y+1}{2}\right)^2$

$x = \frac{1}{4}(y+1)^2 + 1.$

parabola opening right with vertex $(1,-1)$.

$\vec{r}'(t) = 2t\vec{\imath} + 2\vec{\jmath}$

$\vec{r}'(0) = 2\vec{\jmath}$ $\quad |\vec{r}'(0)| = 2$

$\vec{r}'(1) = 2\vec{\imath} + 2\vec{\jmath}$

$|\vec{r}'(1)| = 2\sqrt{2}$

$\vec{r}'(-1) = -2\vec{\imath} + 2\vec{\jmath}$

$|\vec{r}'(-1)| = 2\sqrt{2}$

$|\vec{r}'(t)| = (4t^2 + 4)^{1/2} = 2\sqrt{t^2+1}$

$x = \cos 2t$

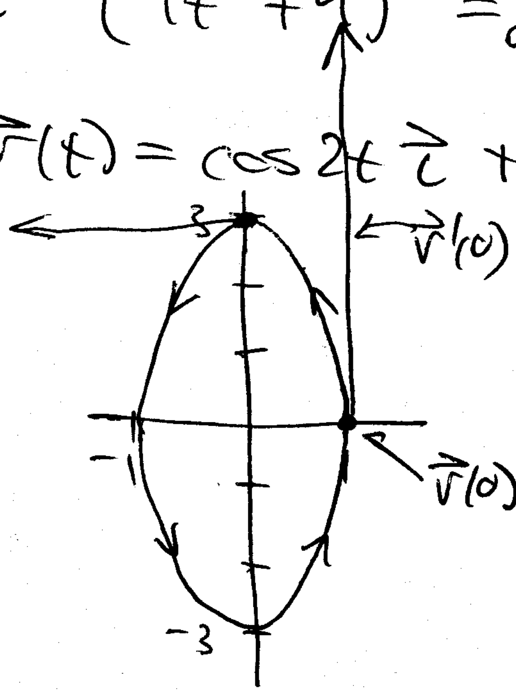
$y = 3\sin 2t$

$\frac{y}{3} = \sin 2t.$

<u>e.g.</u> $\vec{r}(t) = \cos 2t\,\vec{\imath} + 3\sin 2t\,\vec{\jmath}$

$x^2 + \left(\frac{y}{3}\right)^2 = 1$ ellipse

Path:



$\vec{r}'(t) = -2\sin(2t)\vec{\imath} + 6\cos(2t)\vec{\jmath}$

$\vec{r}'(0) = 6\vec{\jmath}$

$|\vec{r}'(t)| = \left(4\sin^2(2t) + 36\cos^2(2t)\right)^{1/2}$

$= \left(4\sin^2(2t) + 4\cos^2(2t) + 32\cos^2(2t)\right)^{1/2}$

$= \left(4 + 32\cos^2(2t)\right)^{1/2}$

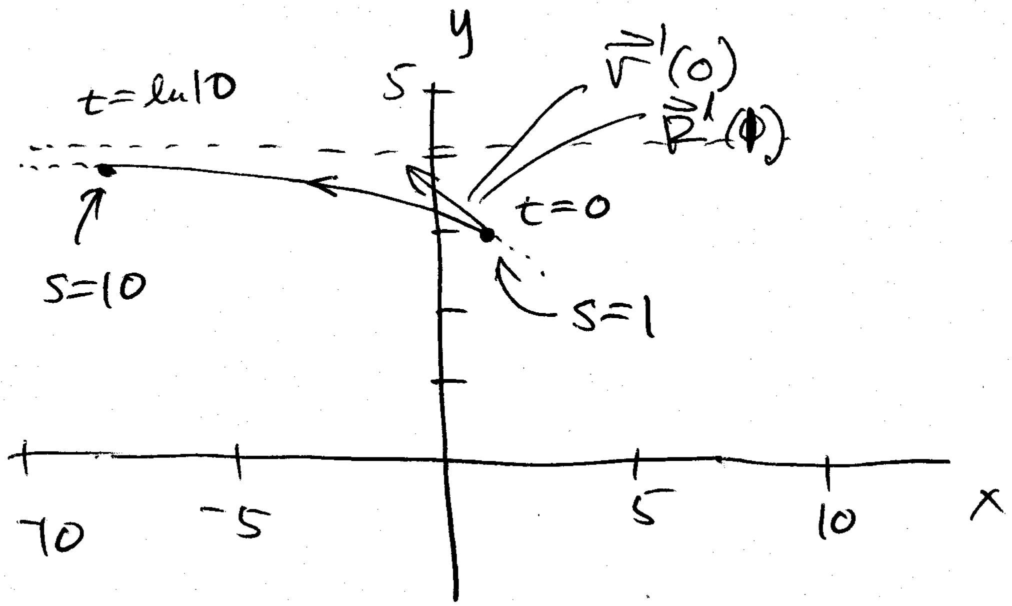
$= 2(1+8\cos^2(2t))^{1/2}$ $\quad |\vec{r}'(\frac{\pi}{4})| = 2$

$\vec{r}(\frac{\pi}{4}) = 3\vec{j}$

11.7 Motion in Space.

#16) $\vec{r}(t) = \langle 2-e^t, 4-e^{-t}\rangle$ $\quad t\in[0,\ln 10]$

$\vec{R}(s) = \langle 2-s, 4-1/s\rangle$ $\quad s\in[1,10]$



$\vec{r}(0) = \langle 1,3\rangle$

$\vec{r}(\ln 10) = \langle -8, 3.9\rangle$

$t=0$ $\quad \vec{r}(0) = \langle 1,3\rangle$

Solve

$\langle 2-s, 4-1/s\rangle = \langle 1,3\rangle$

$\underline{s=1}$

$t=\ln 10$ $\quad \vec{r}(\ln 10) = \langle -8, 3.9\rangle$

$\langle 2-s, 4-1/s\rangle = \langle -8, 3.9\rangle$

$\underline{s=10}$

In fact: $\vec{r}(t) = \vec{R}(e^t)$

so $\underline{\underline{s=e^t}}$ $\quad \underline{\underline{t=\ln s}}$

$\vec{r}\,'(t) = \langle -e^{t}, e^{-t} \rangle \quad |\vec{r}\,'(t)| = (e^{2t} + e^{-2t})^{1/2}$

$\vec{R}\,'(s) = \langle -1, 1/s^2 \rangle \quad |\vec{R}\,'(s)| = \left(1 + \frac{1}{s^4}\right)^{1/2}$

#28) $\vec{a}(t) = \langle e^{-t}, 1 \rangle \quad \langle u_0, v_0 \rangle = \langle 1, 0 \rangle$ initial velocity

Find $\vec{r}(t)$. $\quad \langle x_0, y_0 \rangle = \langle 0, 0 \rangle$ initial position

$\vec{r}\,''(t) = \langle e^{-t}, 1 \rangle$

$\vec{r}\,'(t) = \langle -e^{-t} + c_1, t + c_2 \rangle = \langle -e^{-t}, t \rangle + \overbrace{\langle c_1, c_2 \rangle}^{\vec{c}}$

$\vec{r}\,'(0) = \langle 1, 0 \rangle = \langle -1, 0 \rangle + \underbrace{\vec{c}}_{\langle c_1, c_2 \rangle} \rightarrow \vec{c} = \langle 2, 0 \rangle$

$\vec{r}\,'(t) = \langle -e^{-t} + 2, t \rangle$

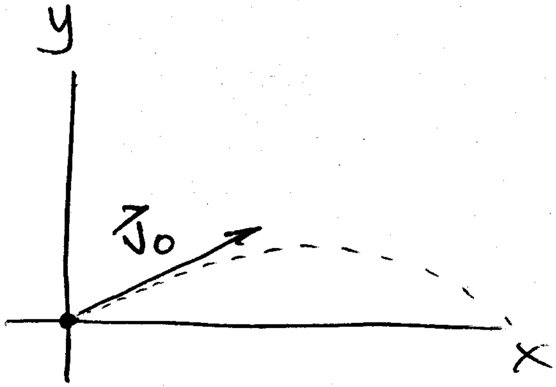
$$\boxed{\begin{array}{l} -1 + c_1 = 1 \\ 0 + c_2 = 0 \end{array} \rightarrow \begin{array}{l} c_1 = 2 \\ c_2 = 0 \end{array}}$$

$\vec{r}(t) = \langle e^{-t} + 2t, \tfrac{1}{2}t^2 \rangle + \vec{c}_0$

$\vec{r}(0) = \langle 0, 0 \rangle = \langle 1, 0 \rangle + \vec{c}_0 \rightarrow \vec{c}_0 = \langle -1, 0 \rangle$

$\vec{r}(t) = \langle e^{-t} + 2t - 1, \tfrac{1}{2}t^2 \rangle$ //

30)



$\vec{r}(t)$ = position of ball at time t.

Know: $\vec{r}(0) = \langle 0,0 \rangle = \vec{0}$

$\vec{r}\,'(0) = 150\left(\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$

$\vec{a}(t) = \vec{r}\,''(t) = -32\vec{j}$

$\vec{r}\,'(t) = -32t\vec{j} + \vec{c}_0$

$\vec{r}\,'(0) = 150\left(\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right) = \vec{c}_0$

$\vec{c}_0 = 75\sqrt{3}\,\vec{i} + 75\vec{j}$

$\vec{r}\,'(t) = 75\sqrt{3}\,\vec{i} + (75 - 32t)\vec{j}$

$\vec{r}(t) = 75\sqrt{3}\,t\,\vec{i} + (75t - 16t^2)\vec{j} + \vec{c}_1$

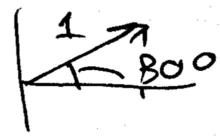
$\vec{r}(0) = \vec{0} = \vec{c}_1$

$$\boxed{\vec{r}(t) = 75\sqrt{3}\,t\,\vec{i} + (75t - 16t^2)\vec{j}}$$

When does ball hit ground?

$75t - 16t^2 = 0$

$t = 0 \quad t = \frac{75}{16} \approx \underline{4.7\text{s}}$.



unit vector $= \cos 30°\,\vec{i} + \sin 30°\,\vec{j}$

$= \frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}$

When is vertical comp. of velocity = 0?

$t = \frac{75}{32} \approx 2.3$ sec.

This is when ball reaches max height.

Max height $\cong$

$75(2.3) - 16(2.3)^2$

$= 87.9$ ft

Range $\cong 75\sqrt{3}(4.7)$

$= 610.5$ ft