Quiz 2 - 11.3, 11.4

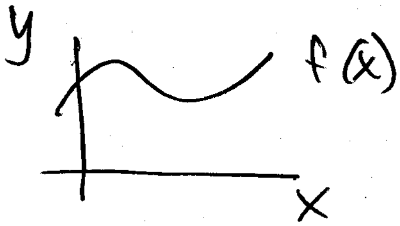
Quiz 1 will be returned on RCT

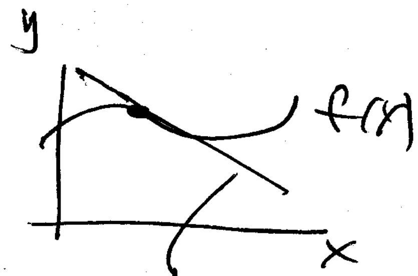
Quiz 1 solutions are on web.

Vector functions.

$y = f(x)$

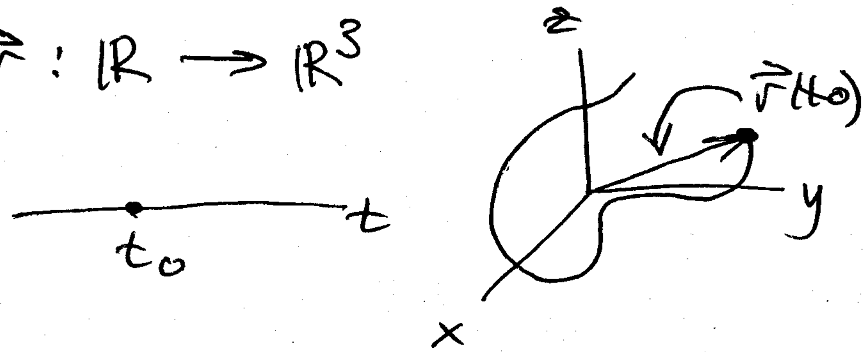
$f: \mathbb{R} \to \mathbb{R}$





$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

$\vec{r}: \mathbb{R} \to \mathbb{R}^3$



$\vec{r}'(t)$? Later.

Lines. $\vec{r}(t) = \vec{OP_0} + t\vec{v}$

P_0 = point on line

$\vec{v}$ = direction vector.

$x = x_0 + tv_1$

$y = y_0 + tv_2$

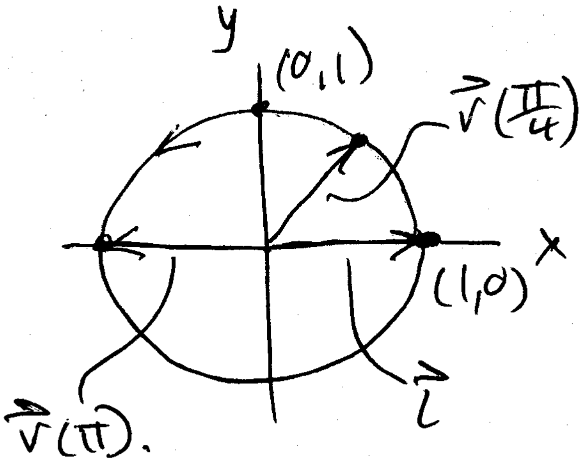
$z = z_0 + tv_3$

$P_0 = (x_0, y_0, z_0)$

$\vec{v} = \langle v_1, v_2, v_3 \rangle$

POINT: Same line can be parametrized differently.

e.g. circular motion



$\vec{r}(t) = \cos t\, \vec{i} + \sin t\, \vec{j}$

$\vec{r}(0) = \vec{i}$

$\vec{r}(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}$

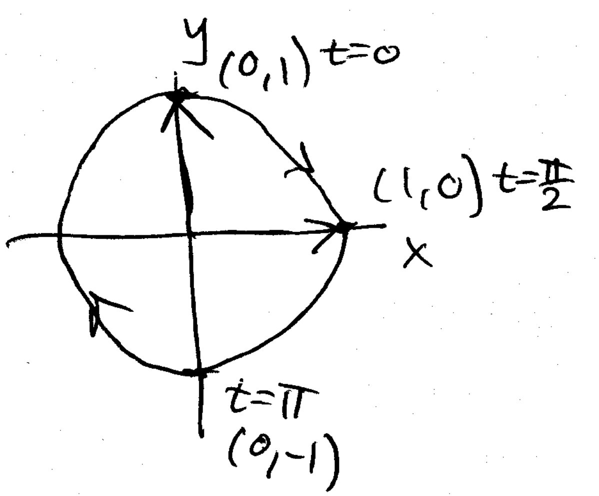
$\vec{r}(\pi) = -\vec{i}$

Note: 1) $|\vec{r}(t)| = (\cos^2 t + \sin^2 t)^{1/2} = 1$ all t.

2) As t increases, $\vec{r}(t)$ "moves" in a counterclockwise direction.

3) There are other ways to ~~par~~ define circular motion.

e.g. $\vec{r}(t) = \sin t\, \vec{i} + \cos t\, \vec{j}$ Still $|\vec{r}(t)| = 1$ all t.



$\vec{r}(0) = \vec{j}$

$\vec{r}(\frac{\pi}{2}) = \vec{i}$

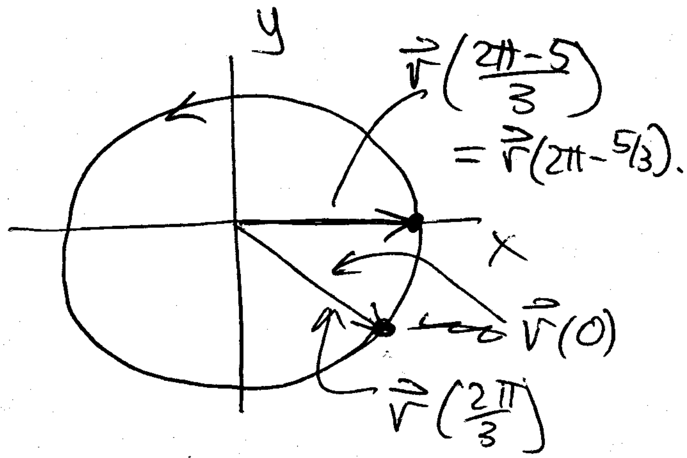
same curve, different function.

e.g. $\vec{r}(t) = 3\sin t\,\vec{i} + 3\cos t\,\vec{j}$

Now $|\vec{r}(t)| = 3$ all t.

e.g. $\vec{r}(t) = \cos(5+3t)\vec{i} + \sin(5+3t)\vec{j}$

still we have $|\vec{r}(t)| = 1$ but



$\vec{r}(0) = \cos 5\,\vec{i} + \sin 5\,\vec{j}$

$\vec{r}(\ ) = \vec{i}$.

need is

$5+3t = 2\pi$

$t = \frac{2\pi-5}{3}$

$\vec{r}(-5/3) = \vec{i}$

when do I return to where I started? $t = 2\pi$? $t = 2\pi - 5/3$? $t = \frac{2\pi}{3}$?

$\vec{r}(2\pi) = \cos(5+6\pi)\vec{i} + \sin(5+6\pi)\vec{j}$

$= \cos(5)\vec{i} + \sin(5)\vec{j}$ so ok.

$\vec{r}(2\pi - 5/3) = \cos(5+3(2\pi-5/3))\vec{i} + \sin(5+3(2\pi-5/3))\vec{j}$

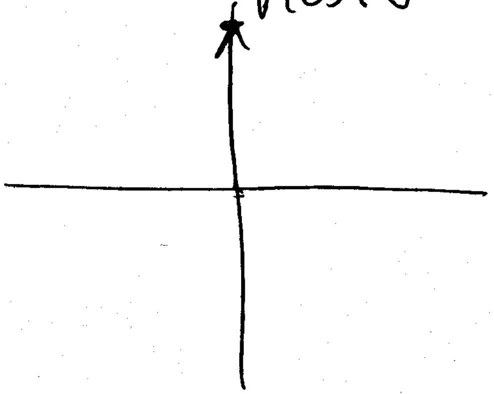
$= \cos(6\pi)\vec{i} + \sin(6\pi)\vec{j} = \vec{i}$

$\vec{r}\left(\frac{2\pi}{3}\right) = \cos\left(5+3\cdot\frac{2\pi}{3}\right)\vec{i} + \sin\left(5+3\cdot\frac{2\pi}{3}\right)\vec{j}$

$= \cos(5+2\pi)\vec{i} + \sin(5+2\pi)\vec{j}$

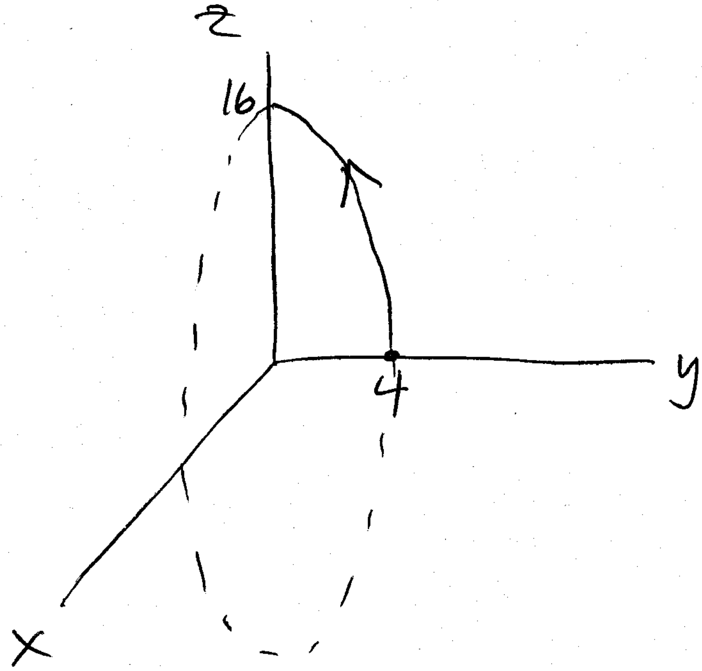
$= \cos(5)\vec{i} + \sin(5)\vec{j}$.

e.g $\sin(t^2)\vec{i} + \cos(t^2)\vec{j} = \vec{r}(t)$.



e.g. #22 (11.5) $\vec{r}(t) = 4\cos t\,\vec{j} + 16\sin t\,\vec{k}$

$0 \le t \le 2\pi$



$\vec{r}(t)$ is in y-z plane.

Elliptical motion!

$y = 4\cos t$

$z = 16\sin t$

$y^2 + \left(\frac{z}{4}\right)^2 =$

$16\cos^2 t + 16\sin^2 t = 16$

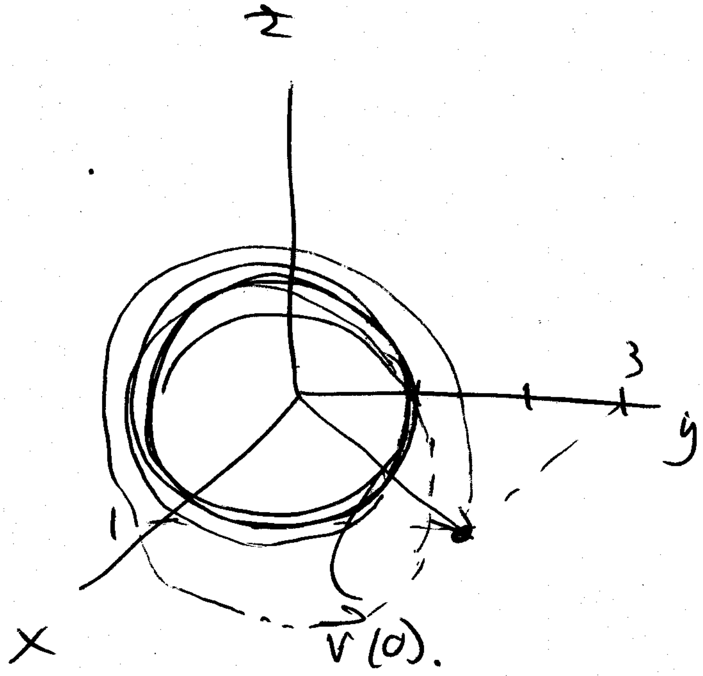
curve is the ellipse

$y^2 + \frac{z^2}{16} = 16$

#28 $\vec{r}(t) = e^{-t}\vec{i} + 3\cos t\,\vec{j} + 3\sin t\,\vec{k}$

$0 \le t < \infty$.

$\vec{r}(0) = \vec{i} + 3\vec{j}$.



11.6 Calculus of Vector Functions

1. Since $\vec{r}(t)$ is a vector, all vector operations can be applied to $\vec{r}(t)$.
2. Taking limits is done componentwise.

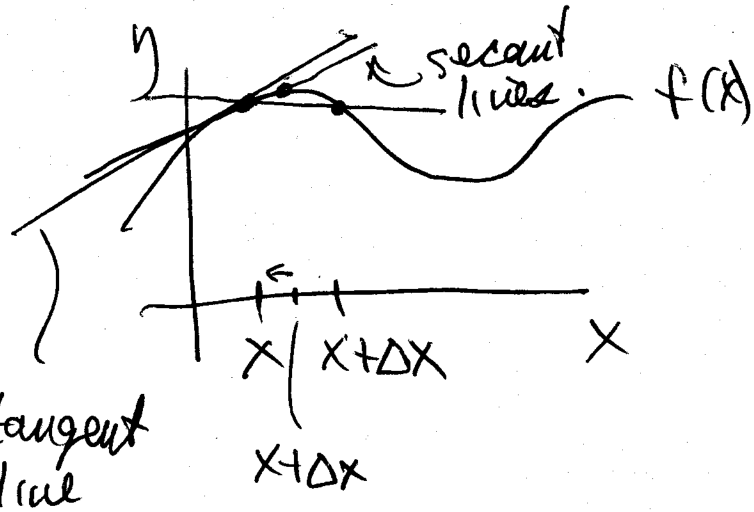
$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

$$\lim_{t\to a} \vec{r}(t) = \left(\lim_{t\to a} x(t)\right)\vec{i} + \left(\lim_{t\to a} y(t)\right)\vec{j} + \left(\lim_{t\to a} z(t)\right)\vec{k}$$

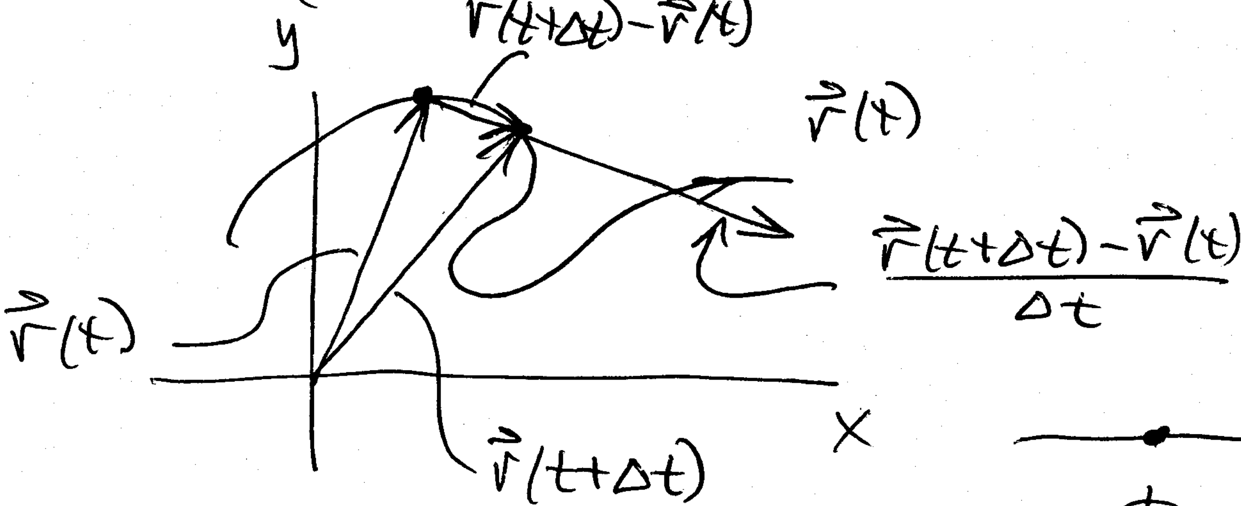
Derivative of $\vec{r}(t)$?

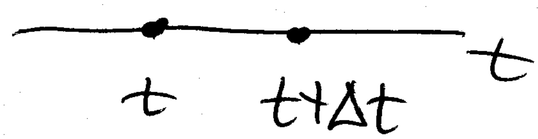
$$y = f(x)$$

$$y' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



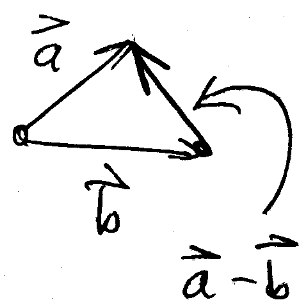
$\vec{r}(t)$ (assume 2-D for sketch).

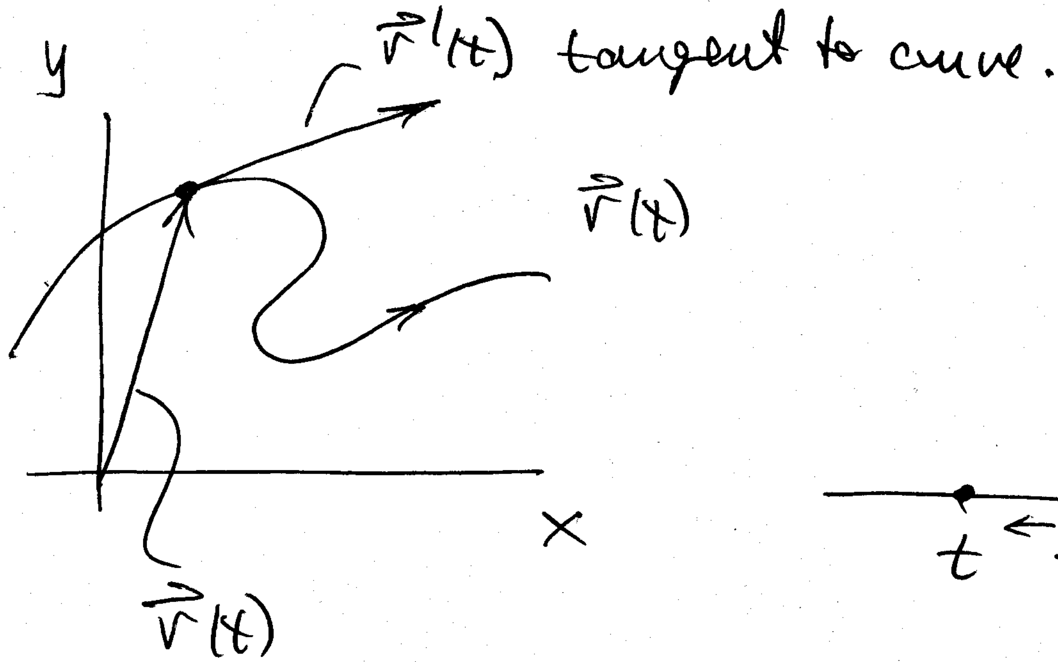


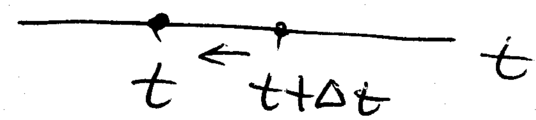


$$\vec{r}\,'(t) = \lim_{\Delta t \to 0} \underbrace{\underbrace{\frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}}_{\text{has direction of secant line}}}_{\text{direction of the tangent line.}}$$

$= \text{tangent vector}$



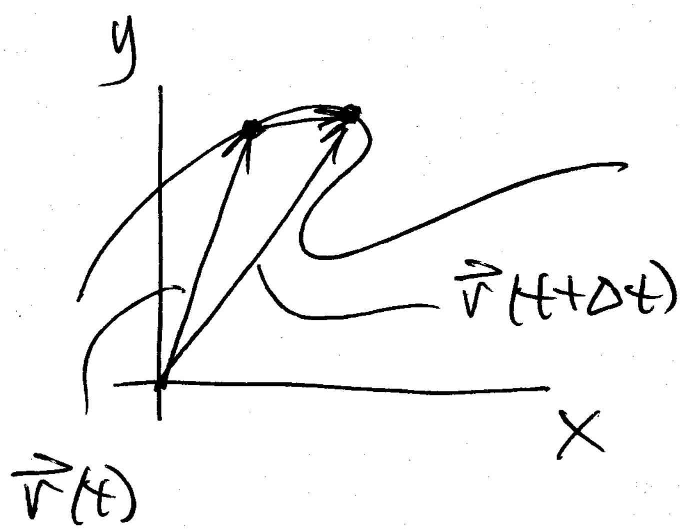




What is $|\vec{r}\,'(t)|$.

$$|\vec{r}\,'(t)| = \lim_{\Delta t \to 0} \frac{|\vec{r}(t+\Delta t) - \vec{r}(t)|}{\Delta t}$$

displacement per unit time = average <u>speed</u>.



$|\vec{r}(t+\Delta t) - \vec{r}(t)|$ = displacement between t and $t+\Delta t$.

$|\vec{r}\,'(t)|$ = rate of change of ~~position~~ displacement at t

= <u>speed</u> at t.