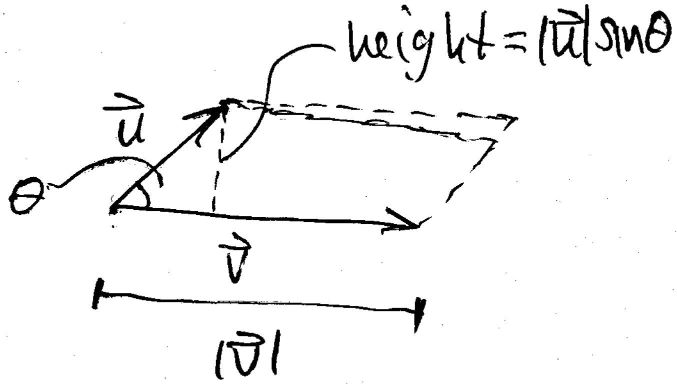
Quizzes will be returned in RCT.

Cross product: $\vec{u} \times \vec{v}$

1. $\vec{u} \times \vec{v}$ is perpendicular to $\vec{u}$ and $\vec{v}$ (direction of $\vec{u} \times \vec{v}$ follows "right-hand rule")
2. $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta$, $0 \le \theta \le \pi$ is angle between $\vec{u}$ and $\vec{v}$.
3. $|\vec{u} \times \vec{v}|$ is the area of parallelogram formed by $\vec{u}$ and $\vec{v}$.



Area = (base) (height)

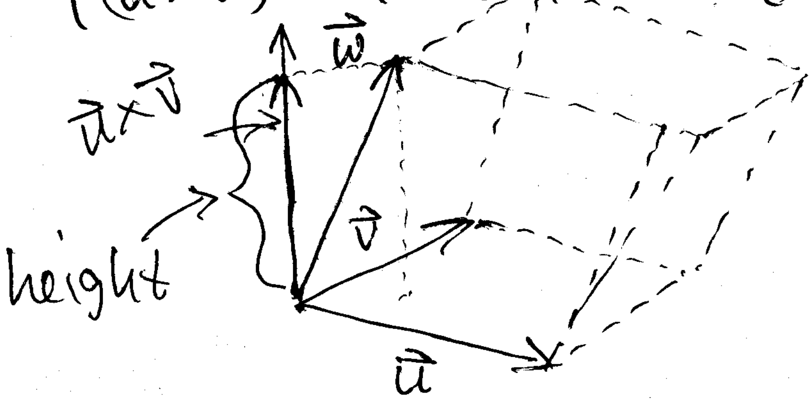
$= (|\vec{v}|)(|\vec{u}|\sin\theta)$

$= |\vec{u}||\vec{v}|\sin\theta$

$= |\vec{u} \times \vec{v}|.$

4. $\vec{u}, \vec{v}, \vec{w}$ $\qquad (\vec{u} \times \vec{v}) \cdot \vec{w} \leftarrow$ vector triple product.

$|(\vec{u} \times \vec{v}) \cdot \vec{w}| =$ volume of box formed by $\vec{u}, \vec{v}, \vec{w}$



volume = (area of base) $\times$ (height)

$$\text{height} = \left|\operatorname{scal}_{\vec{u}\times\vec{v}}(\vec{w})\right| = \left|\frac{(\vec{u}\times\vec{v})\cdot\vec{w}}{|\vec{u}\times\vec{v}|}\right|$$

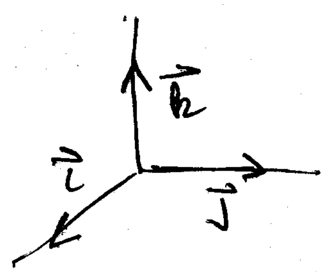
$$\text{Volume} = \left(\cancel{|\vec{u}\times\vec{v}|}\right)\left(\frac{|(\vec{u}\times\vec{v})\cdot\vec{w}|}{\cancel{|\vec{u}\times\vec{v}|}}\right) = |(\vec{u}\times\vec{v})\cdot\vec{w}|.$$

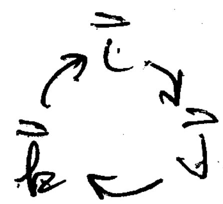
Calculating $\vec{u}\times\vec{v}$ via unit vectors.

<u>e.g.</u> $\vec{u} = 2\vec{i} + \vec{j} + \vec{k}$ $\quad \vec{v} = -4\vec{i} + 3\vec{j} + \vec{k}$

$$\vec{u}\times\vec{v} = -2\vec{i} - 6\vec{j} + 10\vec{k}.$$

$$\vec{i}\times\vec{j} = \vec{k}$$
$$\vec{j}\times\vec{k} = \vec{i}$$
$$\vec{i}\times\vec{k} = -\vec{j}$$





$$\begin{aligned}
&(2\vec{i}+\vec{j}+\vec{k})\times(-4\vec{i}+3\vec{j}+\vec{k}) \\
&= \overset{0}{\cancel{(2\vec{i}\times(-4\vec{i}))}} + (2\vec{i}\times 3\vec{j}) + (2\vec{i}\times\vec{k}) \\
&\quad + (\vec{j}\times(-4\vec{i})) + \overset{0}{\cancel{(\vec{j}\times 3\vec{j})}} + (\vec{j}\times\vec{k}) \\
&\quad + (\vec{k}\times(-4\vec{i})) + (\vec{k}\times 3\vec{j}) + \overset{0}{\cancel{(\vec{k}\times\vec{k})}} \\
&= 6(\vec{i}\times\vec{j}) + 2(\vec{i}\times\vec{k}) - 4(\vec{j}\times\vec{i}) + (\vec{j}\times\vec{k}) - 4(\vec{k}\times\vec{i}) \\
&\quad + 3(\vec{k}\times\vec{j})
\end{aligned}$$

$= 6\vec{k} - 2\vec{j} + 4\vec{k} + \vec{i} - 4\vec{j} - 3\vec{i}$

$= -2\vec{i} - 6\vec{j} + 10\vec{k}$

$$\vec{i} \times \vec{j} = \langle 1,0,0 \rangle \times \langle 0,1,0 \rangle$$

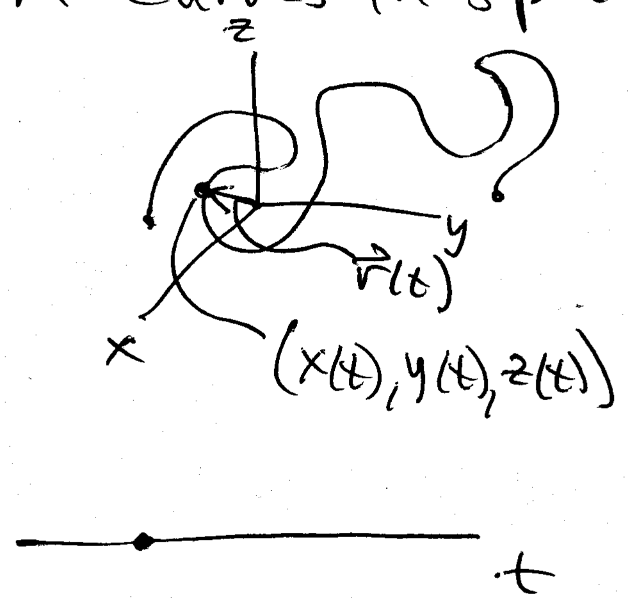
$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

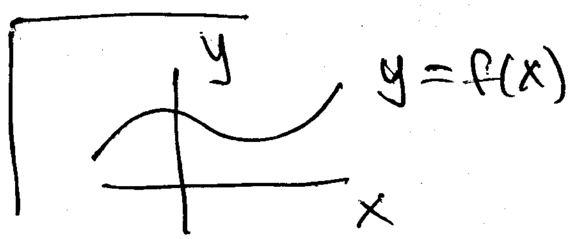
$$\qquad\qquad\qquad 0 \qquad\qquad 0 \qquad\qquad 1$$

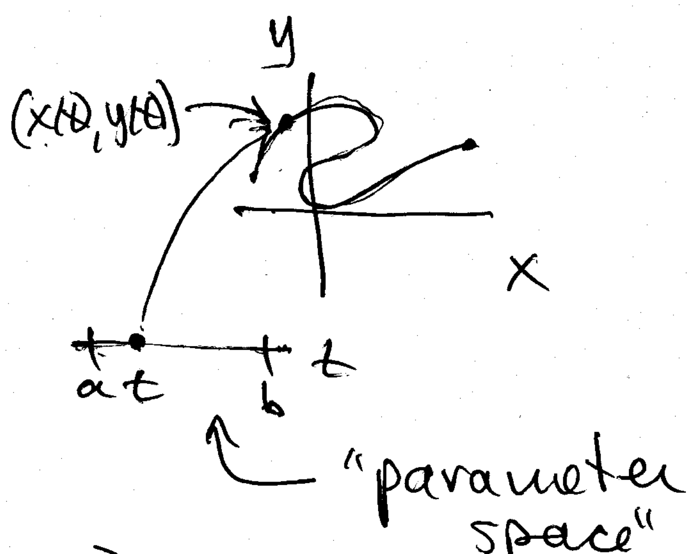
$$= \vec{k}$$

11.5 Lines and Curves.

A. Curves in space.







$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

$\vec{r}(t)$ = vector-valued function of t.

= vector function.

$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$
$= \langle x(t), y(t), z(t) \rangle$

is a vector function that maps $\mathbb{R} \to \mathbb{R}^3$, i.e. it takes numbers to vectors in 3-dimensions.

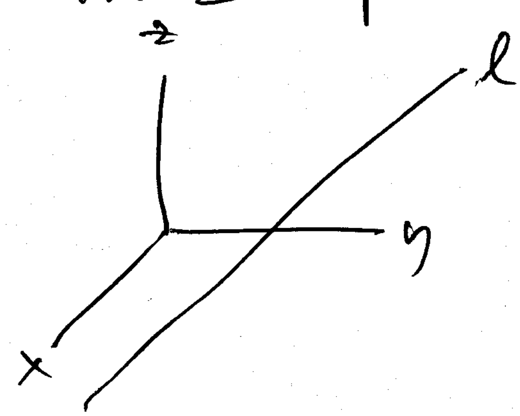
$x(t), y(t), z(t)$ are ordinary functions called the component functions of $\vec{r}(t)$.

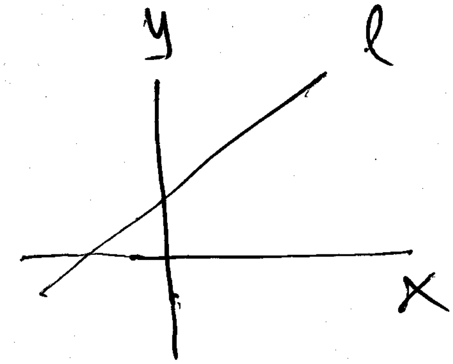
The graph of $\vec{r}(t)$ is the parametric curve

$$\begin{aligned} x &= x(t) \\ y &= y(t) \qquad t \in \mathbb{R}. \\ z &= z(t) \end{aligned}$$

B. Lines in 3-space.

Idea:

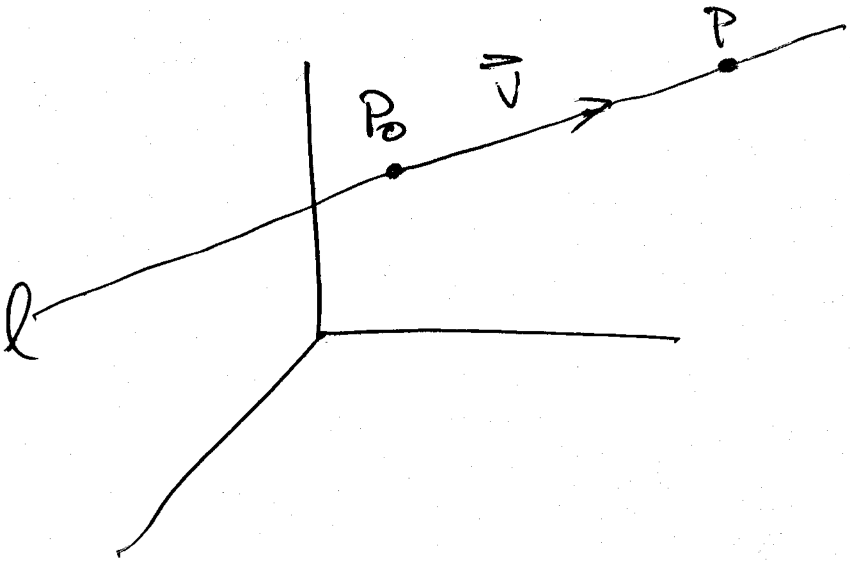




in 2-d need
1) point
2) slope.
not enough in 3-D.

in 3-d you need
1) point
2) direction
(this means unit vector, or just a vector)

Suppose ℓ contains $P_0 = (x_0, y_0, z_0)$ with direction $\vec{v} = \langle v_1, v_2, v_3 \rangle$



If $P = (x, y, z)$ is on ℓ then $\overrightarrow{P_0P}$ is parallel to $\vec{v}$,

so $\overrightarrow{P_0P} = t\,\vec{v}$ for some $t \in \mathbb{R}$.

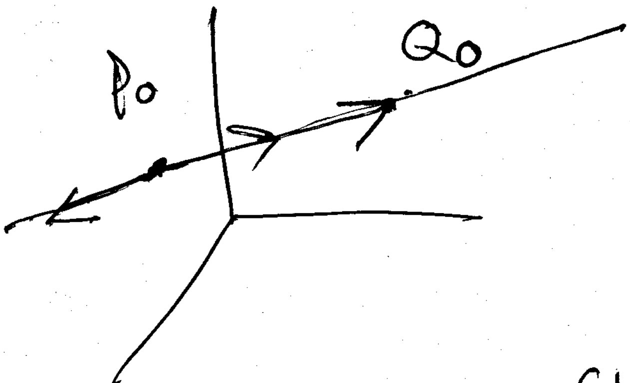
$$\overrightarrow{P_0P} = \langle x - x_0, y - y_0, z - z_0 \rangle \qquad t\vec{v} = \langle tv_1, tv_2, tv_3 \rangle$$

$$\begin{aligned} x - x_0 &= tv_1 \\ y - y_0 &= tv_2 \\ z - z_0 &= tv_3 \end{aligned} \longrightarrow \underbrace{\begin{aligned} x &= x_0 + tv_1 \\ y &= y_0 + tv_2 \\ z &= z_0 + tv_3 \end{aligned}}_{\text{parametric equations of the line } \ell.}$$

Also, ℓ is the graph of

$$\vec{r}(t) = (x_0 + tv_1)\vec{i} + (y_0 + tv_2)\vec{j} + (z_0 + tv_3)\vec{k}$$
$$= \overrightarrow{OP_0} + t\vec{v}.$$

e.g. Find equation of line containing $P_0=(1,2,-1)$ and $Q_0=(-1,0,1)$.



direction vector can be $\overrightarrow{P_0Q_0}$

$\overrightarrow{P_0Q_0}=\langle -2,-2,2\rangle=\vec{v}$

Choose P_0 as the point

Parametric:

$x=1-2t$

$y=2-2t$

$z=-1+2t$

vector function:

$\vec{r}(t)=(1-2t)\vec{i}+(2-2t)\vec{j}+(-1+2t)\vec{k}$

Note: For direction, could use $\langle -1,-1,1\rangle$ or could use $\langle 1,1,-1\rangle$. Also could use Q_0 as my point.

$\vec{v}=\langle -1,-1,1\rangle$ $(-1,0,1)$

$x=-1-s$

$y=0-s$

$z=1+s$

parameter is $s\in\mathbb{R}$

$\langle 1,1,-1\rangle=\vec{v}$ $(1,2,-1)$

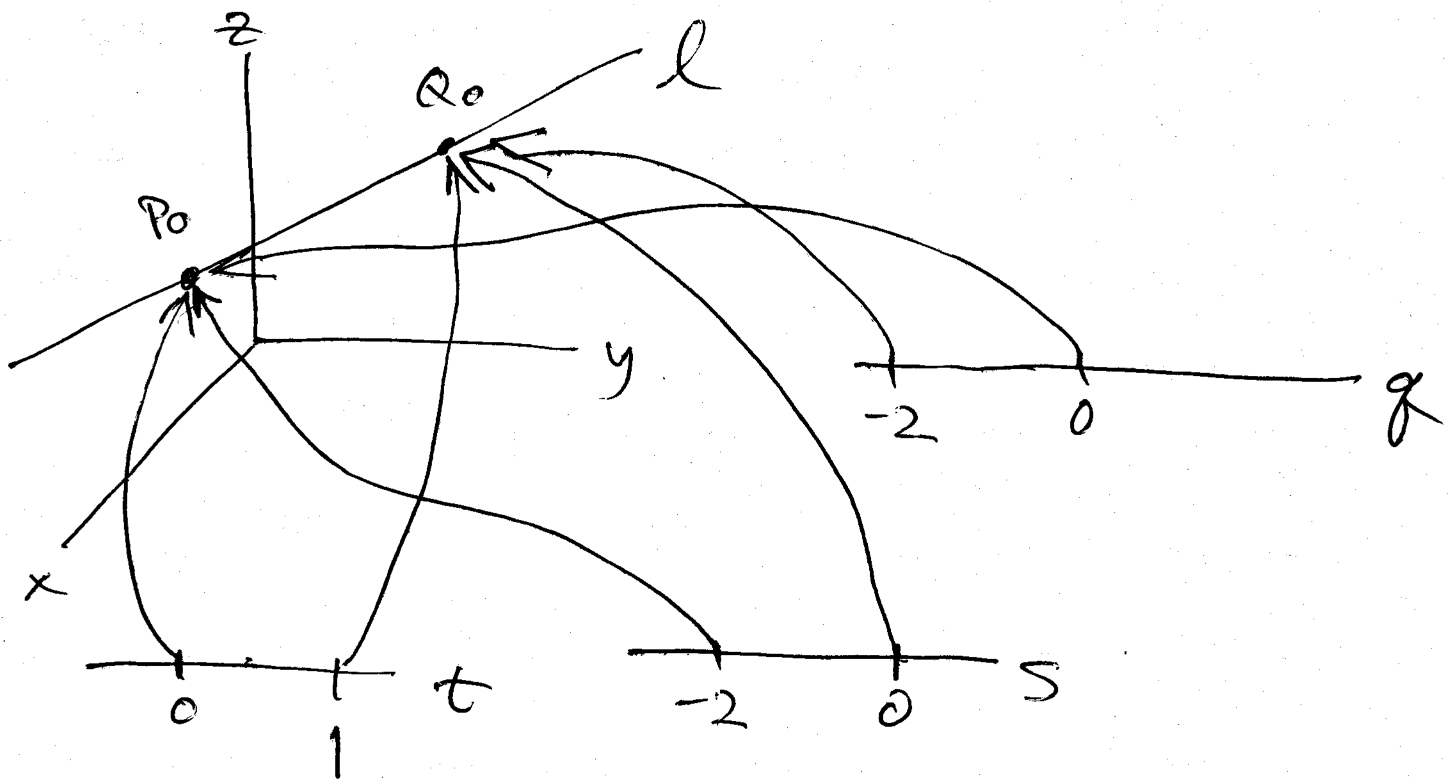
$x=1+g$

$y=2+g$

$z=-1-g$

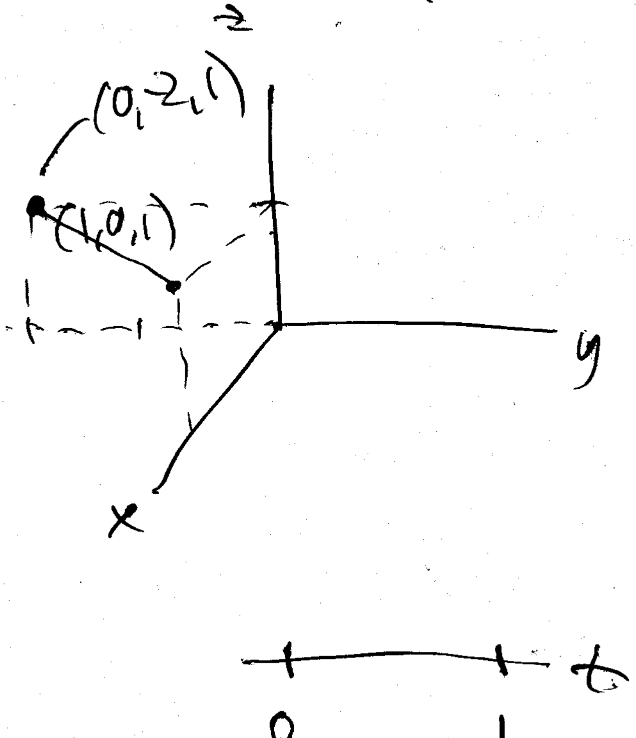
parameter is $g\in\mathbb{R}$.

each gives me the same line.



Line segments.

#18. $(1,0,1)$ $(0,-2,1)$



say we want

$t=0 \longmapsto (1,0,1)$

$t=1 \longmapsto (0,-2,1)$

We can do this by:

$\vec{r}(t) = t\langle 0,-2,1\rangle + (1-t)\langle 1,0,1\rangle$