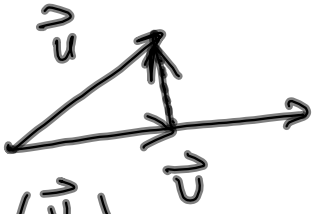
Quiz 1 11.1 – 11.3

Dot product $\langle u_1,u_2,u_3\rangle \cdot \langle v_1,v_2,v_3\rangle = u_1v_1 + u_2v_2 + u_3v_3$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$$

Projection



$\vec{u} = \vec{u}_1 + \vec{u}_2$

$\vec{u}_1$ parallel to $\vec{v}$

$\vec{u}_2$ perp. to $\vec{v}$

$$\text{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\right)\left(\frac{\vec{v}}{|\vec{v}|}\right)$$

$$\text{scal}_{\vec{v}}(\vec{u})$$

e.g. #26 $\vec{u} = \langle 3, -5, 2 \rangle$ $\vec{v} = \langle -9, 5, 1 \rangle$

$$proj_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\right)\left(\frac{\vec{v}}{|\vec{v}|}\right) = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}(\vec{v}) = \frac{-50}{107}\langle -9, 5, 1 \rangle$$

$\vec{u}\cdot\vec{v} = -27 - 25 + 2 = -50$

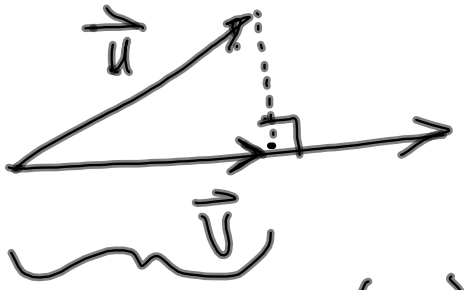
$|\vec{u}| = (9 + 25 + 4)^{1/2} = \sqrt{38}$

$|\vec{v}| = (81 + 25 + 1)^{1/2} = \sqrt{107}$

$$scal_{\vec{v}}(\vec{u}) = \frac{-50}{\sqrt{107}}$$

$$= \frac{-50}{\sqrt{107}}\left(\frac{\langle -9, 5, 1 \rangle}{\sqrt{107}}\right)$$

$scal_{\vec{v}}(\vec{u})$ ↑ unit vector ↑



$|\text{scal}_{\vec{v}}(\vec{w})| = \text{length of } \text{proj}_{\vec{v}}(\vec{w})$

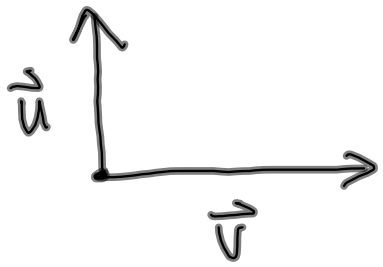
$\text{proj}_{\vec{v}}(\vec{w}) = (\quad)\left(\frac{\vec{v}}{|\vec{v}|}\right)$

$\text{scal}_{\vec{v}}(\vec{w})$

#28 $\vec{u} = \vec{i} + 4\vec{j} + 7\vec{k}$ $\vec{v} = 2\vec{i} - 4\vec{j} + 2\vec{k}$

$\vec{u} \cdot \vec{v} = 2 - 16 + 14 = 0$

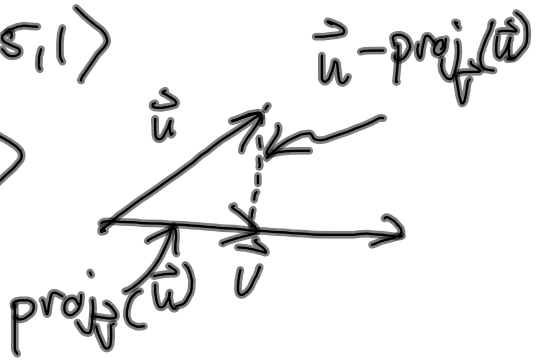
$\text{proj}_{\vec{v}}(\vec{u}) = \vec{0}$



#26 again

$\vec{u} = \langle 3, -5, 2 \rangle$ $\vec{v} = \langle -9, 5, 1 \rangle$

$\text{proj}_{\vec{v}}(\vec{u}) = \frac{-50}{107}\langle -9, 5, 1 \rangle$



$\vec{w} = \vec{u} - \text{proj}_{\vec{v}}(\vec{u})$

$= \langle 3, -5, 2 \rangle - \left(\frac{-50}{107}\right)\langle -9, 5, 1 \rangle$

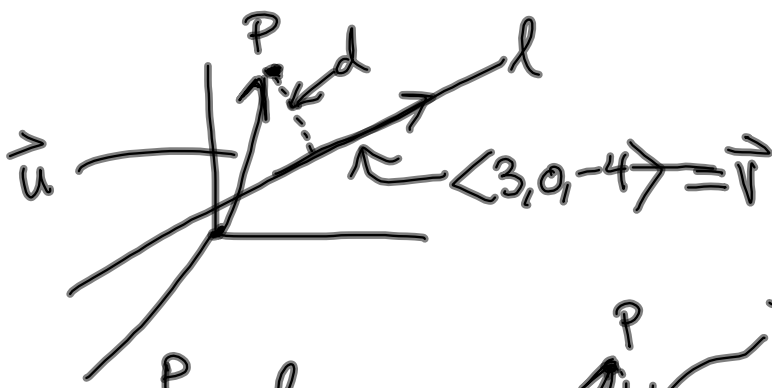
$= \left\langle \frac{-129}{107}, \frac{-285}{107}, \frac{264}{107} \right\rangle$

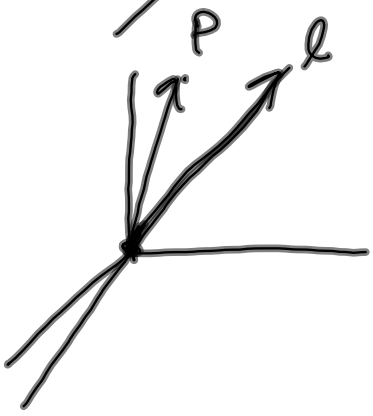
$\vec{w} \cdot \vec{v} = (-9)\left(\frac{-129}{107}\right) - \left(\frac{285}{107}\right)(5) + (1)\left(\frac{264}{107}\right)$

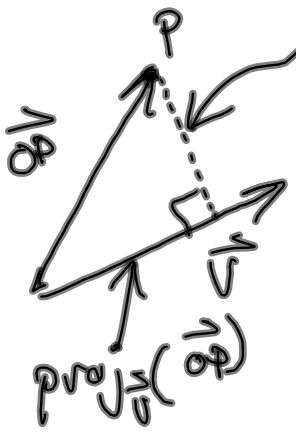
$= \frac{1}{107}(1161 - 1425 + 264)$

$= 0$

e.g. #54 $P=(0,2,6)$ ℓ has direction $\langle 3,0,-4\rangle$







$\vec{w} = \vec{OP} - \text{proj}_{\vec{v}}(\vec{OP})$

$|\vec{w}|$ = dist from point to line

Works because line is through origin.