Syllabus on course web page

Quiz 1 – Tuesday 1-31 11.1 – 11.3

Vectors. $\vec{v} = \langle v_1, v_2 \rangle$ $\vec{v} = \langle v_1, v_2, v_3 \rangle$

Representations of vectors by arrows.

magnitudes of vectors

direction of vectors (unit vectors)

scalar multiplication

Def: Two vectors are parallel if one is a scalar multiple of the other.

3. Vector addition

(done component-wise)

#26, p691 $\vec{u} = \langle 4, -2 \rangle$ $\vec{v} = \langle -4, 6 \rangle$

$$\vec{u} + \vec{v} = \langle 4, -2 \rangle + \langle -4, 6 \rangle = \langle 4 + (-4), -2 + 6 \rangle = \langle 0, 4 \rangle$$

$$2\vec{u} + 3\vec{v} = 2\langle 4, -2 \rangle + 3\langle -4, 6 \rangle = \langle 8, -4 \rangle + \langle -12, 18 \rangle$$
$$= \langle -4, 14 \rangle$$

#36 p701

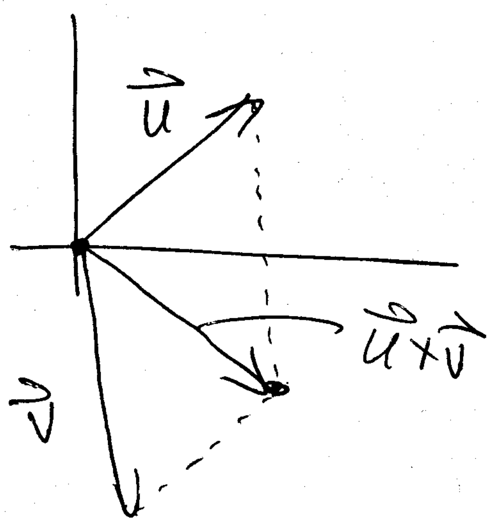
$\vec{u} = \langle -1, 1, 0 \rangle \quad \vec{v} = \langle 2, -4, 1 \rangle$

$\vec{u} + \vec{v} = \langle -1, 1, 0 \rangle + \langle 2, -4, 1 \rangle = \langle 1, -3, 1 \rangle$

$4\vec{u} - \vec{v} = 4\langle -1, 1, 0 \rangle - \langle 2, -4, 1 \rangle$

$= \langle -4, 4, 0 \rangle - \langle 2, -4, 1 \rangle = \langle -6, 8, -1 \rangle$

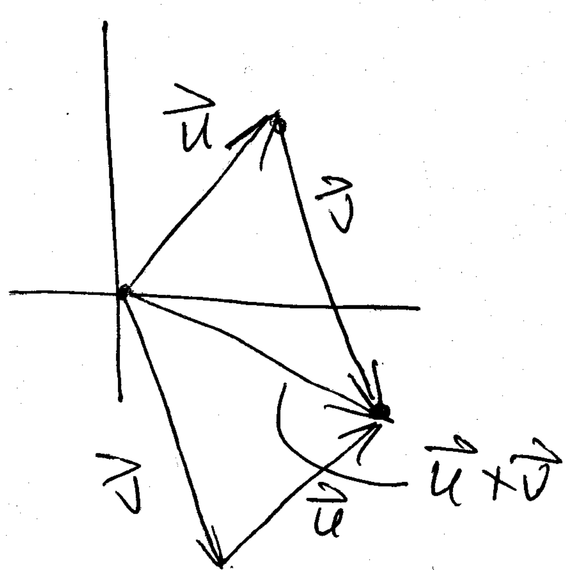
Geometric interpretation.



① Think: Complete the parallelogram formed by $\vec{u}, \vec{v}$. $\vec{u}+\vec{v}$ is the diagonal.

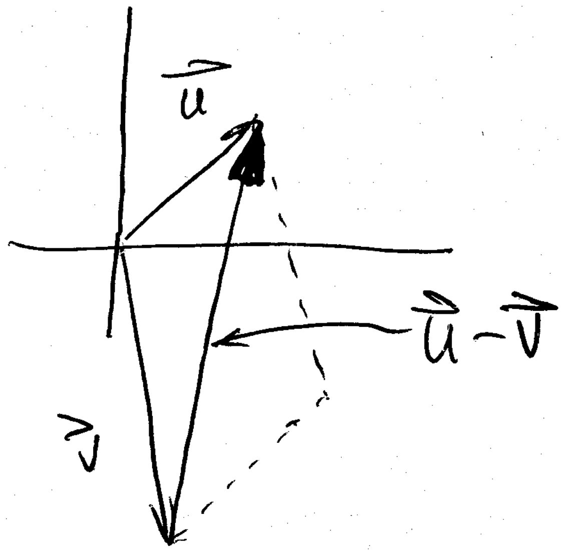
OR

② Think: Total displacement.



#20 p 690

$\overrightarrow{BF} = \vec{u} + 3\vec{v}$ $\quad \overrightarrow{AD} = 2\vec{u} - 3\vec{v}$



What is $\vec{u} - \vec{v}$?

Note: $\vec{v} + (\vec{u} - \vec{v}) = \vec{u}$

4. Standard basis vectors.

$\vec{i} = \langle 1,0,0 \rangle \quad \vec{j} = \langle 0,1,0 \rangle \quad \vec{k} = \langle 0,0,1 \rangle$

(in the plane $\vec{i} = \langle 1,0 \rangle \quad \vec{j} = \langle 0,1 \rangle$).

$\vec{u} = \langle 8,-4 \rangle = \langle 8,0 \rangle + \langle 0,-4 \rangle$

$= 8\langle 1,0 \rangle - 4\langle 0,1 \rangle = 8\vec{i} - 4\vec{j}$

$\vec{v} = \langle 2,4,1 \rangle = \langle 2,0,0 \rangle + \langle 0,4,0 \rangle + \langle 0,0,1 \rangle$

$= 2\langle 1,0,0 \rangle + 4\langle 0,1,0 \rangle + \langle 0,0,1 \rangle$

$= 2\vec{i} + 4\vec{j} + \vec{k}$.

11.3 Dot product (or inner product)

A. Definition:

1. The <u>dot product</u>, <u>inner product</u> or <u>scalar product</u> of $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ is

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$\left(\vec{u} = \langle u_1, u_2 \rangle, \vec{v} = \langle v_1, v_2 \rangle, \vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 \right)$$

<u>e.g.</u> $\vec{u} = \langle 2, -2, 1 \rangle$ $\vec{v} = \langle 3, 4, -2 \rangle$

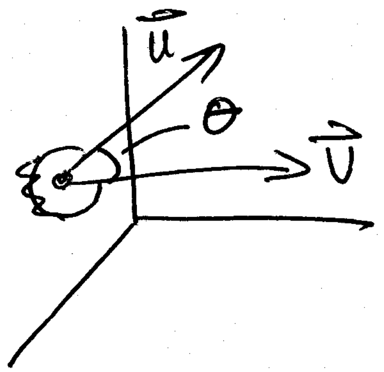
$$\vec{u} \cdot \vec{v} = (2)(3) + (-2)(4) + (1)(-2)$$
$$= 6 - 8 - 2 = -4.$$

Note: $\vec{u}, \vec{v}$ are <u>vectors</u>, $\vec{u} \cdot \vec{v}$ is a <u>scalar</u>.

2. Algebraic Properties.

<u>Thm 11.2</u> In these ways the dot product is similar to ordinary product of numbers.

3. Angle between vectors.



θ = angle between $\vec{u}$ and $\vec{v}$

Always $0 \le \theta \le \pi$.

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$$

This is a restatement of the Law of Cosines.

<u>Note:</u> $\dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|} = \left(\dfrac{\vec{u}}{|\vec{u}|}\right) \cdot \left(\dfrac{\vec{v}}{|\vec{v}|}\right)$

unit vectors
in direction of $\vec{u}, \vec{v}$

<u>e.g.</u> #14 $\vec{u} = \langle 3,4,0 \rangle$ $\vec{v} = \langle 0,4,5 \rangle$

Find $\vec{u} \cdot \vec{v}$ and approximate θ.

$\vec{u} \cdot \vec{v} = 0 + 16 + 0 = 16$

$|\vec{u}| = (9 + 16 + 0)^{1/2} = 25^{1/2} = 5$

$|\vec{v}| = (0 + 16 + 25)^{1/2} = \sqrt{41}$

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|} = \dfrac{16}{5 \cdot \sqrt{41}} \approx .50$ $\quad \theta \approx \dfrac{\pi}{3}$.

#15) $\vec{u} = \vec{i} - 4\vec{j} - 6\vec{k}$ $\quad \vec{v} = 2\vec{i} - 4\vec{j} + 2\vec{k}$

$$\vec{u}\cdot\vec{v} = (1)(2) + (-4)(-4) + (-6)(2)$$
$$= 2 + 16 - 12 = 6$$

$|\vec{u}| = (1+16+36)^{1/2} = \sqrt{53}$ $\quad |\vec{v}| = (4+16+4)^{1/2} = \sqrt{24}$
$= 2\sqrt{6}$

$$\cos\theta = \frac{\cancel{6}^{3}}{\cancel{2}\sqrt{6}\cdot\sqrt{53}} \approx .17 \qquad \theta \cong 80^\circ$$

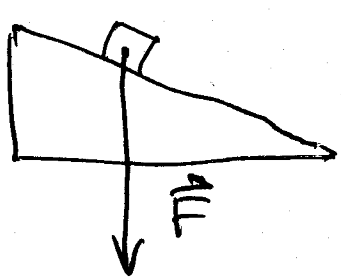
4. Orthogonal vectors.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|} = 0 \Leftrightarrow \vec{u}\cdot\vec{v} = 0$$
$$\Leftrightarrow \theta = 90^\circ = \frac{\pi}{2}$$

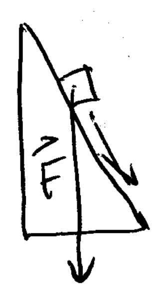
If $\vec{u}\cdot\vec{v} = 0$ we say $\vec{u}$ and $\vec{v}$ are orthogonal.

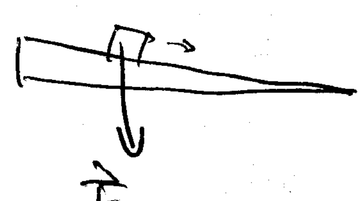
B. Vector Projection

Idea: Consider simple set-up



How hard is object being pulled down ramp?

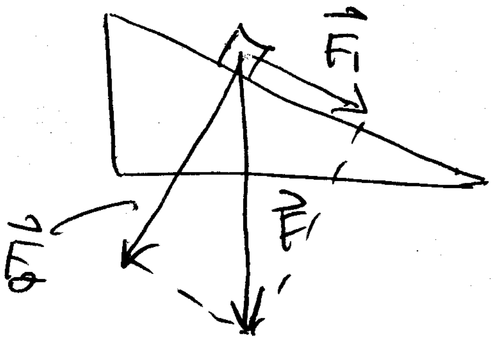




We write $\vec{F} = \vec{F_1} + \vec{F_2}$

$\vec{F_1}$: directed down ramp

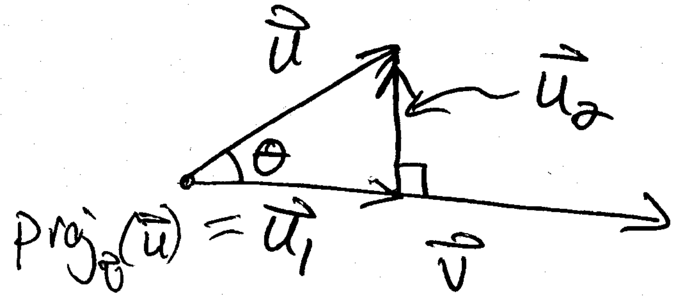
$\vec{F_2}$: directed perpendicular to ramp.



General problem:

Given $\vec{u}$ and $\vec{v}$, write $\vec{u} = \vec{u_1} + \vec{u_2}$ where $\vec{u_1}$ is parallel to $\vec{v}$ and $\vec{u_2}$ is orthogonal to $\vec{v}$.

We write $\vec{u_1} = \text{proj}_{\vec{v}}(\vec{u}) =$ (vector) projection of $\vec{u}$ along $\vec{v}$.



How to find $\vec{u_1}$:

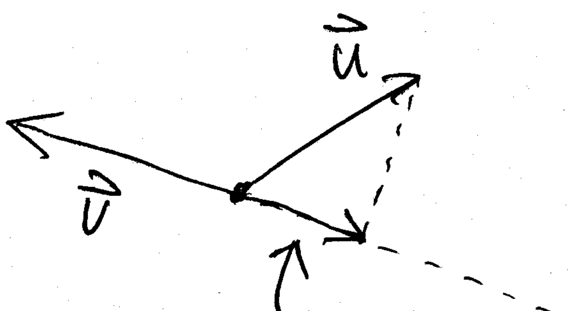
$? = |\vec{u}| \cos\theta$

$$|\vec{u}|\cos\theta = \cancel{|\vec{u}|}\cdot\frac{\vec{u}\cdot\vec{v}}{\cancel{|\vec{u}|}|\vec{v}|} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$$

$$\mathrm{proj}_{\vec{v}}(\vec{u}) = \underbrace{\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\right)}_{\text{"length" (can be } <0)}\underbrace{\left(\frac{\vec{v}}{|\vec{v}|}\right)}_{\text{direction}} = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v}$$

$$\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \mathrm{scal}_{\vec{v}}(\vec{u}) \quad \text{(is a scalar quantity)}$$

It is possible that $\mathrm{scal}_{\vec{v}}(\vec{u}) < 0$.
What does this mean?



$\mathrm{proj}_{\vec{v}}(\vec{u})$ ← will be a negative multiple of $\frac{\vec{v}}{|\vec{v}|}$.

$(\mathrm{scal}_{\vec{v}}(\vec{u}) < 0.)$