11.1/11.2 Vectors

A. Definition

1. Intuition: length/magnitude + direction

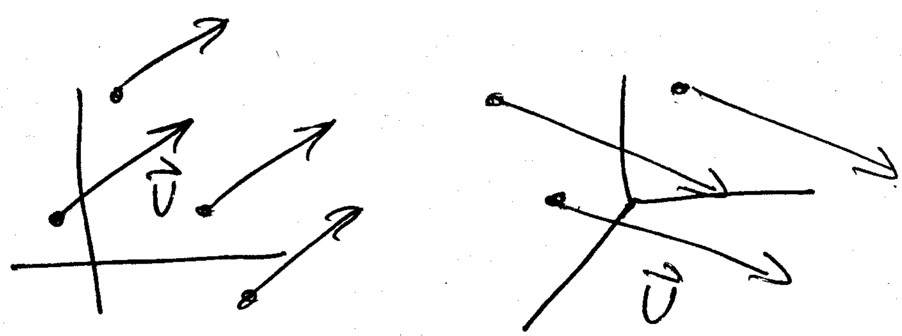
<u>eg</u>. velocity, acceleration, force

2. Mathematical: vector is an <u>ordered pair</u> or <u>triple</u>, written

$\vec{v} = \langle v_1, v_2 \rangle \quad \vec{v} = \langle v_1, v_2, v_3 \rangle$

v_1, v_2, v_3 are <u>components</u> of $\vec{v}$.

3. Geometrical/Graphical: vector is an <u>arrow</u> in the plane or in 3-space

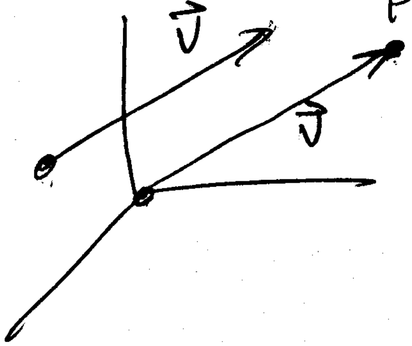


A vector $\vec{v}$ is <u>represented by</u> an arrow.

Any arrow with the same magnitude and direction represents $\vec{v}$. One vector $\vec{v}$: Many representations

B. Computing with vectors

1. Representation (arrow) $\Rightarrow$ vector.



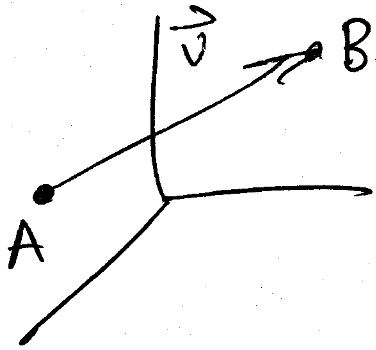
$P = (v_1, v_2, v_3)$

$\vec{v} = \langle v_1, v_2, v_3 \rangle$

a. find a representation of $\vec{v}$ with tail at the origin.

b. If head of vector is at $P = (v_1, v_2, v_3)$, then $\vec{v} = \langle v_1, v_2, v_3 \rangle$.

More specifically



Usually write $\vec{v} = \vec{AB}$

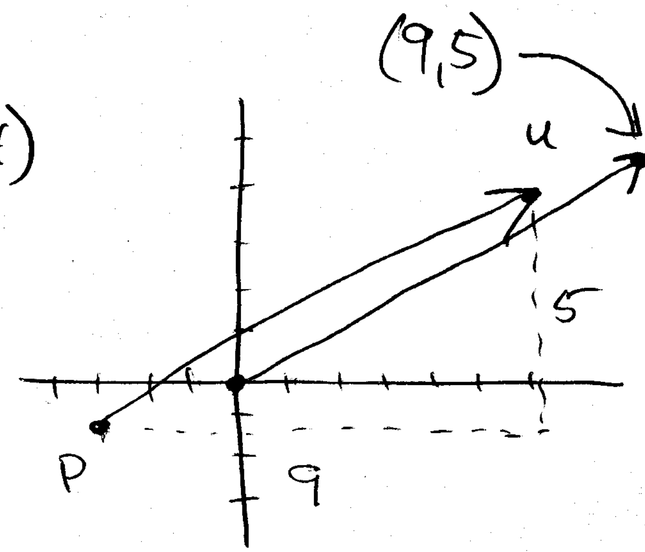
If $A = (x_1, y_1, z_1)$
$B = (x_2, y_2, z_2)$ then

$\vec{v} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

eg #22 p 691.

$\vec{PU}$ $P = (-3, -1)$, $U = (6, 4)$

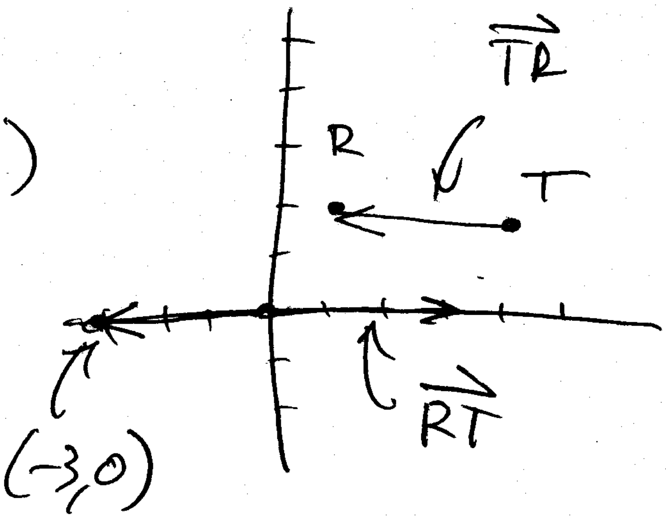
$\vec{PU} = \langle 9, 5 \rangle$



$\overrightarrow{TR} = \langle -3, 0 \rangle$

$T = (4,2) \quad R = (1,2)$

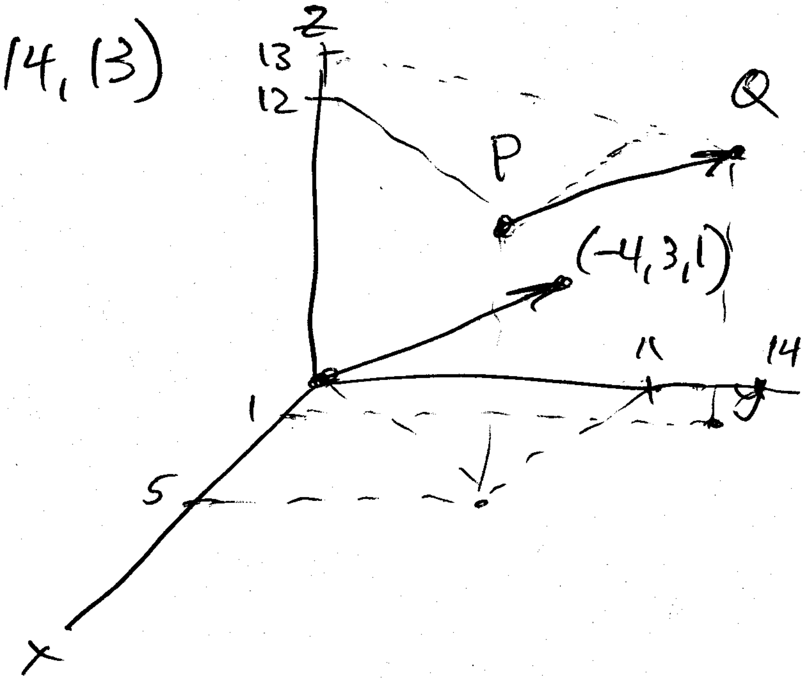
$\overrightarrow{RT} = \langle 3, 0 \rangle$



#40 p 701

$P = (5, 11, 12) \quad Q = (1, 14, 13)$

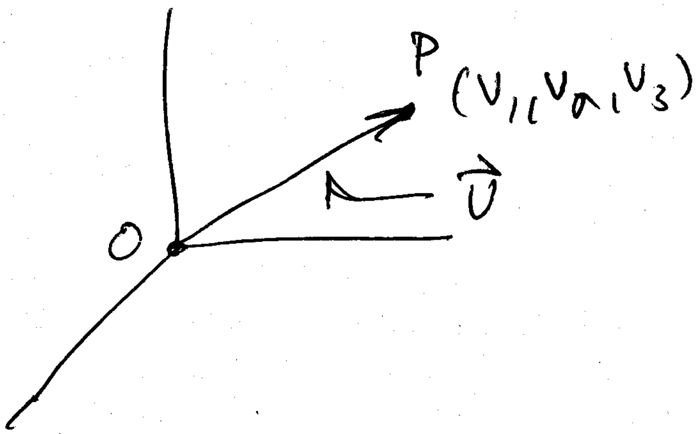
$\overrightarrow{PQ} = \langle -4, 3, 1 \rangle$



2. vector (ordered triple (pair)) $\longrightarrow$ representation (arrow)

$\vec{v} = \langle v_1, v_2, v_3 \rangle \quad P = (v_1, v_2, v_3)$

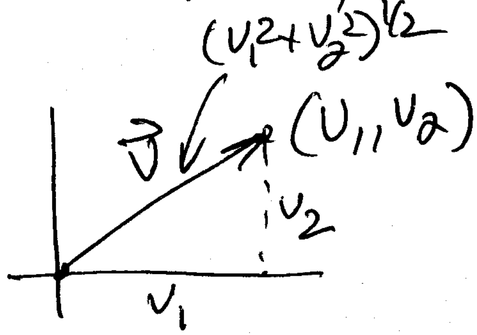
The arrow $\overrightarrow{OP}$ is a representation of $\vec{v}$ and is called the <u>position vector</u> of $\vec{v}$



3. Magnitude of $\vec{v}$

$\vec{v} = \langle v_1, v_2, v_3 \rangle, \quad |\vec{v}| = (v_1^2 + v_2^2 + v_3^2)^{1/2}$

$\vec{v} = \langle v_1, v_2 \rangle, \quad |\vec{v}| = (v_1^2 + v_2^2)^{1/2}$



$|\vec{v}|$ = length of any arrow representing $\vec{v}$.

eg: $|\vec{PQ}| = |\langle -4, 3, 1 \rangle| = (16 + 9 + 1)^{1/2}$
$= (26)^{1/2} = \sqrt{26}.$

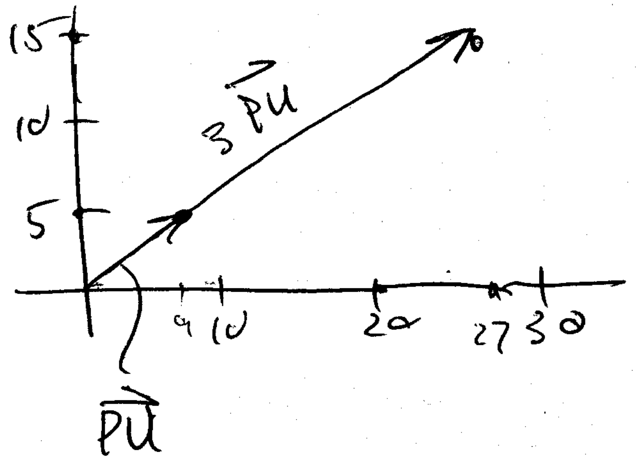
C. Vector operations.

1. scalar multiplication (done componentwise)

#22 p 691

$\overrightarrow{PU} = \langle 9, 5 \rangle$

What is $3(\overrightarrow{PU}) = 3\langle 9,5 \rangle = \langle 27, 15 \rangle$

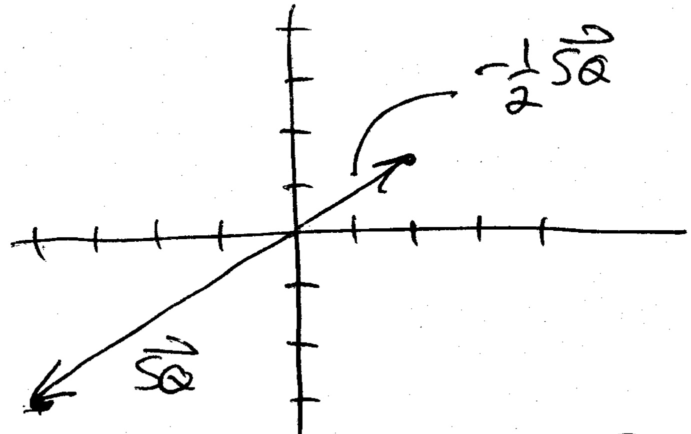


$\overrightarrow{SQ} = \langle -4, -3 \rangle$

$S = (3,5) \quad Q = (-1,2)$

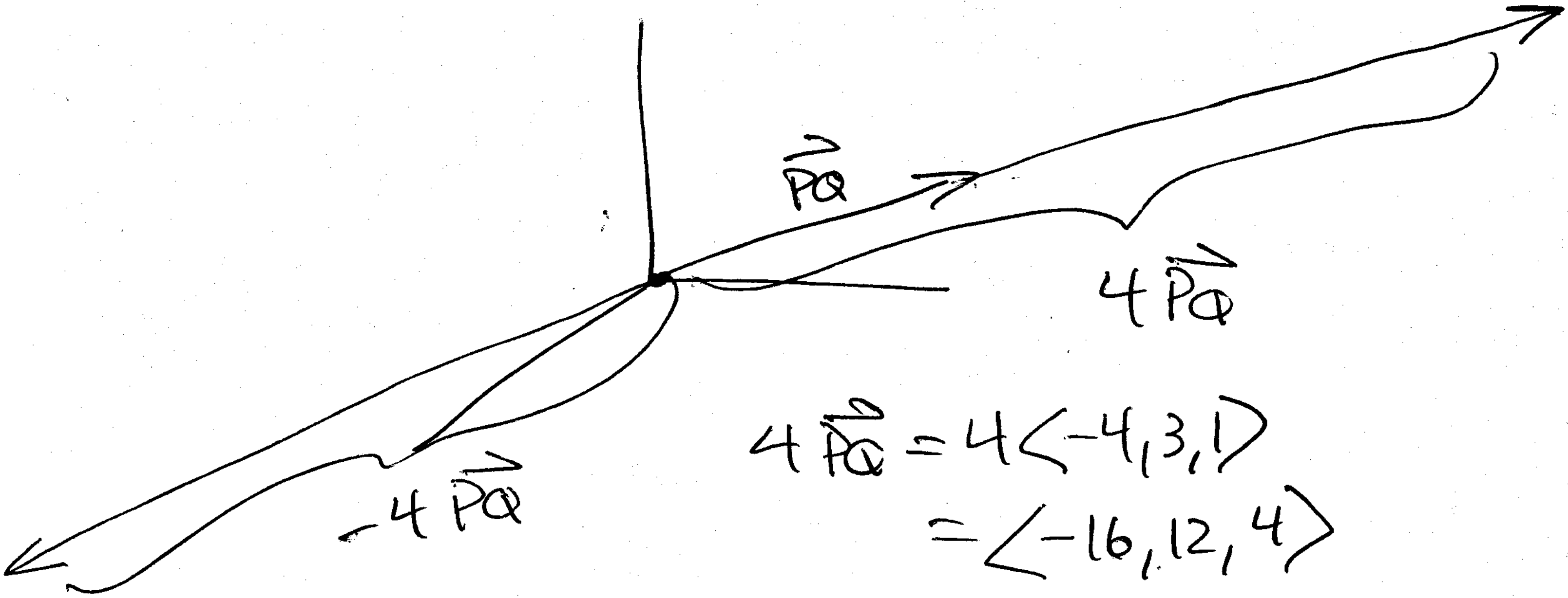
$-\frac{1}{2}\overrightarrow{SQ} = -\frac{1}{2}\langle -4, -3 \rangle$

$= \langle 2, \frac{3}{2} \rangle$



$\vec{PQ} = \langle -4, 3, 1 \rangle$

Find 2 vectors parallel to $\vec{PQ}$ with 4 times the magnitude.



$$4\vec{PQ} = 4\langle -4,3,1\rangle = \langle -16, 12, 4\rangle$$

$$|\vec{PQ}| = \sqrt{26}$$

$$|4\vec{PQ}| = |\langle -16,12,4\rangle| = (256+144+16)^{1/2} = (416)^{1/2} = (4\cdot 104)^{1/2} = (16\cdot 26)^{1/2} = 4\sqrt{26}$$

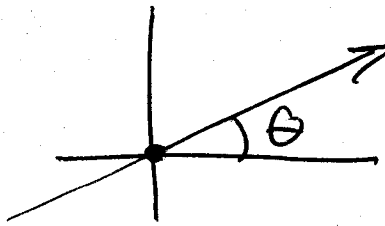
$$-4\vec{PQ} = -4\langle -4,3,1\rangle = \langle 16,-12,-4\rangle$$

$|-4\vec{PQ}| = 4\sqrt{26}$. In general $|\alpha \vec{v}| = |\alpha|\,|\vec{v}|$

scalar   abs. value

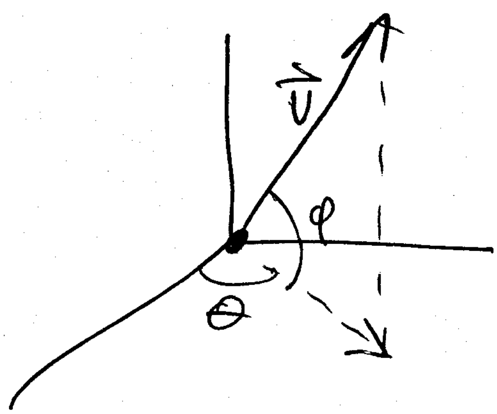
2. Unit Vector

Idea: gives a precise notion of direction in any dimension.



θ describes the direction

slope also will describe direction



Describing direction is more complicated

A unit vector is a vector with length 1.

Given $\vec{v}$, the unit vector in the direction of $\vec{v}$ is $\dfrac{\vec{v}}{|\vec{v}|} = \left(\dfrac{1}{|\vec{v}|}\right)\vec{v}$

$$\left|\left(\frac{1}{|\vec{v}|}\right)\vec{v}\right| = \frac{1}{|\vec{v}|}|\vec{v}| = 1$$

eg $\vec{PQ} = \langle -4, 3, 1\rangle$ $\quad \dfrac{\vec{PQ}}{|\vec{PQ}|} = \dfrac{1}{\sqrt{26}}\langle -4, 3, 1\rangle$

$$= \left\langle \frac{-4}{\sqrt{26}}, \frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}} \right\rangle$$

Any vector $\vec{v}$ can be written as

$$\vec{v} = (\text{magnitude}) \cdot (\text{direction})$$

$$= \underbrace{|\vec{v}|}_{\text{magnitude}} \underbrace{\left(\frac{\vec{v}}{|\vec{v}|}\right)}_{\text{direction.}}$$