MATH 213 – 5 MAY 2004 – FINAL EXAM

Answer each of the following questions. Show all work, as partial credit may be given. This exam will be counted out of a total of 120 points.

1. (10 pts.) Two resistors have resistances R_1 and R_2 respectively. When connected in parallel, the total resistance is given by $R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$. If R_1 is given as 300 ohms with a maximum error of 15 ohms and R_2 is given as 600 ohms with a maximum error of 6 ohms, *use differentials* to estimate the maximum error in the computed resistance R.

2. Suppose that a projectile is fired from the ground with an initial speed of 300 feet per second at an angle of elevation of 60 degrees.

(a) (8 pts.) Find the initial velocity, $\mathbf{v}_0$ of the projectile.

(b) (6 pts.) Find an expression for $\mathbf{r}(t)$, the position of the particle t seconds after it has been fired. (Hint: $\mathbf{r}(t) = -16\,t^2\,\mathbf{j} + \mathbf{v}_0\,t + \mathbf{r}_0$, where $\mathbf{v}_0$ is the initial velocity, and $\mathbf{r}_0$ the initial position of the projectile.)

(c) (6 pts.) At what time t does the projectile strike the ground? How far away from its initial position does it strike the ground?

3. (8 pts. each) Let $f(x, y) = x^2 + 2y^2 + x^2y + 4$.

(a) Find all critical points of f. (Hint: There are three.)

(b) Determine whether each of the critical points you found in part (a) is local maximum, local minimum, or saddle point.

4. Consider the integral $\iint_D (x^2 + xy)\, dA$ where D is the region in the x-y plane bounded by the curves $y = x^2$ and $y = 4$.

(a) (8 pts.) Write the given integral as an iterated integral in two different ways. Do not evaluate.

(b) (6 pts.) Evaluate one of the iterated integrals you found in part (a).

5. (12 pts.) Compute the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}$ is the vector field $\mathbf{F} = x^2\,\mathbf{i} + xy\,\mathbf{j} - z^2\,\mathbf{k}$, and C is the line segment from the point $(0, 2, 1)$ to the point $(4, 2, 5)$.

6. (12 pts.) Set up but do not evaluate the line integral $\int_C x\,y^2\,z^3\,ds$ where C is the curve given by $\mathbf{r}(t) = 2\,\cos(t)\,\mathbf{i} + 2\,\sin(t)\,\mathbf{j} + t^2\,\mathbf{k}$, $0 \le t \le 2\pi$.

7. (a) (12 pts.) Let $\mathbf{F} = (8xy + y)\,\mathbf{i} + (4x^2 + x + 2y)\,\mathbf{j}$. Find $f(x, y)$ so that $\mathbf{F} = \nabla f$.

   (b) (8 pts.) Let $\mathbf{F} = (x\,y^3 + z^2)\,\mathbf{i} + (x^2 + y^2 + z^2)\,\mathbf{j} + (x\,y\,z)\,\mathbf{k}$. Compute curl($\mathbf{F}$). is $\mathbf{F}$ conservative? Why or why not? (Hint: For any f, curl$(\nabla f) = 0$.)

   (c) (8 pts.) Find div($\mathbf{F}$) where $\mathbf{F}$ is the vector field given in part (b).

8. (12 pts.) Use Green's theorem to evaluate the line integral $\int_C xy^2\,dx + 2x^2y\,dy$ where C consists of line segments from $(-1, 0)$ to $(1, 0)$, from $(1, 0)$ to $(0, 1)$ and from $(0, 1)$ to $(-1, 0)$.