MATH 213 – FINAL EXAMINATION – 14 DECEMBER 2009

Answer all of the following questions on the answer sheets provided. Do no work on this sheet. Show all work, as partial credit may be given. This exam is counted out of a total of 100 points.

1. (6 pts. each)

(a) Find the vector projection of $\langle -1, 0, 1\rangle$ along $\langle 1, 2, 3\rangle$.

(b) Find parametric equations for the line containing $(-1, 0, 1)$ and $(1, 2, 3)$.

(c) Find the area of the parallelogram formed by the vectors $\langle 0, 1, 1\rangle$ and $\langle 1, 2, 2\rangle$.

2. (6 pts. each) A projectile is fired from the ground, an initial speed of 500 meters per second and an angle of elevation $30°$. Find

(a) $\mathbf{r}(t)$, the vector valued function giving the position of the projectile at time t, and

(b) the range of the projectile.

(Hint: the acceleration due to gravity on the earth is 9.8 meters per second squared.)

3. (4 pts. each) Consider the function $f(x, y, z) = xy^2 + yz^3$.

(a) Find the gradient of f.

(b) In what direction does $f(x, y, z)$ increase most rapidly at the point $(1, -2, 1)$? (I want you to give a unit vector that points in that direction.)

(c) Find the directional derivative of f at the point $(1, -2, 1)$ in the direction of the vector $\langle -1, 1, -1\rangle$.

(d) Find the linearization of f at $(1, -2, 1)$.

(e) Find the equation of the plane tangent to the level surface $xy^2 + yz^3 = 2$ at the point $(1, -2, 1)$.

4. (6 pts. each) Consider the function $f(x, y) = x^2 + 2y^2 + x^2y$.

(a) Find all critical points of f. (There are three.)

(b) Use the second derivative test to identify each of the critical points as a local maximum, local minimum, or a saddle point. Show all work!

5. (6 pts. each)

(a) Evaluate the iterated integral $\int_0^4 \int_{y/2}^2 xy\,dx\,dy$.

(b) Reverse the order of integration in the above integral. Do not evaluate.

6. (6 pts.) Find the Jacobian of the transformation $x = \sin(uv)$, $y = u^2 + v^3$.

7. (6 pts. each) Compute the following.

(a) $\int_C x\,ds$ where C is parametrized by $x = t^3$, $y = t$, $0 \le t \le 1$.

(b) $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the line segment from $(0,1)$ to $(3,0)$, and $\mathbf{F}(x,y) = x^2y\mathbf{i} + 3x^2y^2\mathbf{j}$.

8. (8 pts.) Use Green's Theorem to evaluate the line integral $\int_C 2xy\,dx + xy^2\,dy$ where C consists of the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$, the line segment joining $(1,1)$ to $(0,1)$, and the line segment joining $(0,1)$ to $(0,0)$.