MATH 213 – QUIZ 13 – 1 MAY 2008

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

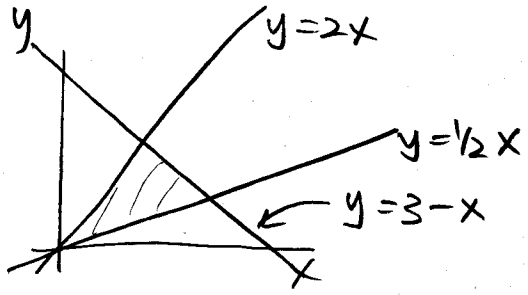
1. (4 pts.) Find the Jacobian of the transformation $x = u/v$, $y = (u+v)^{1/2}$.

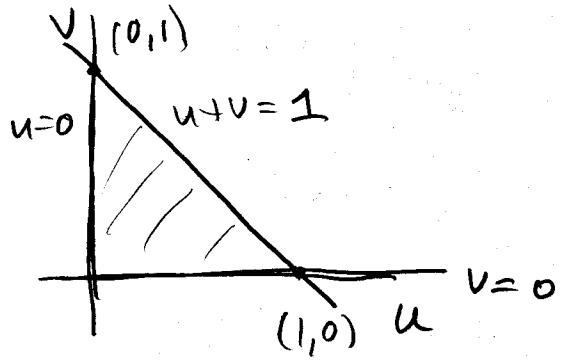
$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{v} & \frac{1}{2}(u+v)^{-1/2} \\ -\frac{u}{v^2} & \frac{1}{2}(u+v)^{-1/2} \end{vmatrix}$$

$$= \left(\frac{1}{v}\right)\left(\frac{1}{2}(u+v)^{-1/2}\right) + \left(\frac{u}{v^2}\right)\frac{1}{2}(u+v)^{-1/2} = \frac{1}{2}(u+v)^{-1/2}\left(\frac{1}{v} + \frac{u}{v^2}\right)$$

$$= 1/2\,\frac{(u+v)^{1/2}}{v^2} \;//$$

2. (6 pts.) Consider the double integral $\iint_R (x - 3y)\,dA$ where R is the region in the x-y plane bounded by the lines $y = 2x$, $y = x/2$, and $y = 3 - x$. Set up but do not evaluate an iterated integral in the variables u and v that is equivalent to the above integral by making the change of variables $x = 2u + v$, $y = u + 2v$ in the above integral. (Hint: The Jacobian of the given transformation is 3).





$y = 2x \longleftrightarrow u + 2v = 4u + 2v$

$u = 0$

$y = \frac{1}{2}x \longleftrightarrow u + 2v = u + \frac{1}{2}v$

$v = 0$

$y = 3 - x \longleftrightarrow u + 2v = 3 - 2u - v$

$3u + 3v = 3$

$u + v = 1$

$$\iint_R (x - 3y)\,dA$$

$$= \int_0^1 \int_0^{1-u} (2u + v - 3u - 6v)(3)\,dv\,du$$

$$= \int_0^1 \int_0^{1-u} -3(u + 5v)\,dv\,du \;//$$