MATH 213 – QUIZ 12 – 24 APRIL 2008

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

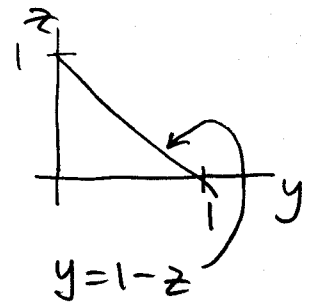
1. (6 pts.) Evaluate the triple integral $\int_0^1 \int_0^{1-z} \int_0^2 xyz\,dx\,dy\,dz$

$$\int_0^1 \int_0^{1-z} \int_0^2 xyz\,dx\,dy\,dz = \int_0^1 \int_0^{1-z} \tfrac{1}{2}x^2yz\Big|_0^2\,dy\,dz$$

$$= \int_0^1 \int_0^{1-z} 2yz\,dy\,dz = \int_0^1 y^2z\Big|_0^{1-z}\,dz = \int_0^1 z(1-z)^2\,dz$$

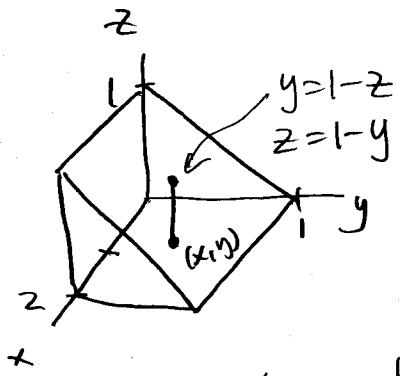
$$= \int_0^1 z - 2z^2 + z^3\,dz = \tfrac{1}{2}z^2 - \tfrac{2}{3}z^3 + \tfrac{1}{4}z^4\Big|_0^1 = \tfrac{1}{2} - \tfrac{2}{3} + \tfrac{1}{4} = \tfrac{1}{12}$$

2. (4 pts.) Change the order of integration in the integral in Problem 1 so that the differential reads $dz\,dy\,dx$. Do not evaluate.



$$\int_0^1 \int_0^{1-z} \int_0^2 xyz\,dx\,dy\,dz$$

$$= \int_0^2 \int_0^1 \int_0^{1-y} (xyz)\,dz\,dy\,dx$$



project onto x-y plane: