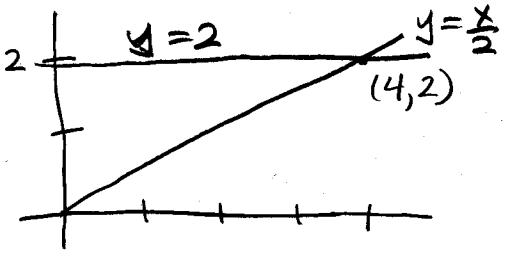
MATH 213 – QUIZ 11 – 17 APRIL 2008

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (4 pts.) Find the mass of the triangular region bounded by the y–axis, the line $y = 2$ and the line $y = x/2$ if the density is given by $\rho(x, y) = y^2$.



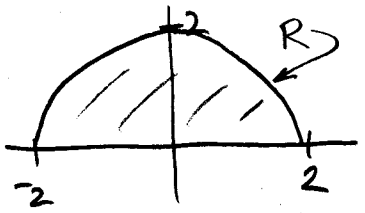
$$M = \iint_R \rho(x,y)\,dA = \int_0^4 \int_{\frac{x}{2}}^2 y^2\,dy\,dx$$

$$= \int_0^4 \tfrac{1}{3}y^3\Big|_{\frac{x}{2}}^2 dx = \int_0^4 \left(\tfrac{8}{3} - \tfrac{1}{24}x^3\right)dx$$

$$= \tfrac{8}{3}x - \tfrac{1}{96}x^4\Big|_0^4 = \tfrac{32}{3} - \tfrac{256}{96} = \tfrac{32}{3} - \tfrac{8}{3} = \tfrac{24}{3} = 8 \,//$$

[Equivalent integral: $\int_0^2 \int_0^{2y} y^2\,dx\,dy = \int_0^2 2y^3\,dy = 8$]

2. (6 pts.) Evaluate the double integral $\iint_R x\,dA$ where R is the upper half of the disk of radius 2 centered at the origin by first converting the integral into polar coordinates.



$0 \le r \le 2$
$0 \le \theta \le \pi$

$$\iint_R x\,dA = \int_0^\pi \int_0^2 r\cos\theta\, r\,dr\,d\theta$$

$$= \int_0^\pi \int_0^2 r^2\cos\theta\,dr\,d\theta = \int_0^\pi \left[\tfrac{1}{3}r^3\cos\theta\Big|_{r=0}^2\right]d\theta$$

$$= \tfrac{8}{3}\int_0^\pi \cos\theta\,d\theta = \tfrac{8}{3}\left(\sin\theta\Big|_0^\pi\right)$$

$$= \tfrac{8}{3}(0-0) = 0 \,//.$$