MATH 213 – QUIZ 9 – 3 APRIL 2008

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (4 pts.) Use differentials to estimate the change in the function $f(x,y,z) = x^3 + 3x^2y + z^3$ if the point $(1,2,-1)$ moves 0.1 units in the direction $3\mathbf{i} + 4\mathbf{k}$?

$\Delta f \approx df = D_{\vec{u}} f(1,2,-1)\,(0.1) = (\nabla f(1,2,-1) \cdot \vec{u})(0.1)$

$= \left[15\left(\frac{3}{5}\right) + 3(0) + 3\left(\frac{4}{5}\right)\right](0.1)$

$= \left(9 + \frac{12}{5}\right)\left(\frac{1}{10}\right) = \frac{57}{50} = 1.14$ //

$\vec{u} = \frac{3\vec{i} + 4\vec{k}}{(9+16)^{1/2}} = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{k}$

$\nabla f = 3x^2 + 6xy\,\vec{i} + 3x^2\vec{j} + 3z^2\vec{k}$

$\nabla f(1,2,-1) = 15\vec{i} + 3\vec{j} + 3\vec{k}$

2. (6 pts.) Find all critical points of the function $f(x,y) = x^3 - y^3 - 6xy + 2$ and determine if each point is a local maximum, local minimum, or saddle point. (Hint: There are two critical points to check.)

CP: $f_x = 3x^2 - 6y$

$f_y = -3y^2 - 6x$

$3x^2 - 6y = 0$
$-3y^2 - 6x = 0$ $\Big\} \rightarrow$ $x^2 - 2y = 0 \rightarrow y = \frac{1}{2}x^2$
$y^2 + 2x = 0$ ←

$\left(\frac{1}{2}x^2\right)^2 + 2x = 0$

$\frac{1}{4}x^4 + 2x = 0$

$x^4 + 8x = 0$

$x(x^3 + 8) = 0$

$x = 0 \quad x = -2$

$y = 0 \quad y = 2 \quad (\text{since } y = \frac{1}{2}x^2)$

$(0,0)\ (-2,2)$ are the C.P. //

$f_{xx} = 6x \quad f_{xy} = -6$

$f_{yy} = -6y$

$D(x,y) = \begin{vmatrix} 6x & -6 \\ -6 & -6y \end{vmatrix}$

$= -36xy - 36$

$(0,0)$: $D(0,0) = -36 < 0$

$\Rightarrow (0,0)$ is a saddle //

$(-2,2)$: $D(-2,2) = -36(-4) - 36 > 0$

$f_{xx}(-2,2) = -12 < 0$

$\Rightarrow (-2,2)$ is a local maximum //