MATH 213 – QUIZ 7 – 20 MARCH 2008

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. Consider the function $f(x, y, z) = xy + yz^2 + xz^3$.

(a) (4 pts.) Find the gradient of f.

$$\nabla f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k}$$

$$= (y + z^3)\vec{i} + (x + z^2)\vec{j} + (2yz + 3xz^2)\vec{k} \;//$$

(b) (3 pts.) Find the directional derivative of f at the point $(2, 0, 3)$ and in the direction $\mathbf{v} = \langle 2, -1, 2 \rangle$.

$$D_{\vec{u}} f(2,0,3) = \nabla f(2,0,3) \cdot \vec{u}$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 2, -1, 2 \rangle}{(4+1+4)^{1/2}} = \left\langle \frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \right\rangle$$

$$\nabla f(2,0,3) = 27\vec{i} + 11\vec{j} + 54\vec{k}$$

$$\therefore D_{\vec{u}} f(2,0,3) = 27 \cdot \frac{2}{3} - 11 \cdot \frac{1}{3} + 54 \cdot \frac{2}{3} = \frac{151}{3} \;//$$

(c) (3 pts.) Find the maximum rate of change of f at the point $(2, 0, 3)$.

$$\text{Max rate of change} = |\nabla f(2,0,3)|$$

$$= (27^2 + 11^2 + 54^2)^{1/2} \;//$$

$$= (3766)^{1/2} \approx 61.4 \;//$$