

MATH 213 - QUIZ 4 - 21 FEBRUARY 2008

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Find the arclength parameter $s(t)$ along the curve $\mathbf{r}(t) = t\mathbf{i} + (2/3)t^{3/2}\mathbf{j} - 6t\mathbf{k}$ starting from the point where $t = 0$.

$$\vec{r}'(t) = \vec{i} + t^{1/2}\vec{j} - 6\vec{k}$$

$$|\vec{r}'(t)| = (1 + t + 36)^{1/2} = (t + 37)^{1/2}$$

$$\begin{aligned} s(t) &= \int_0^t (\tau + 37)^{1/2} d\tau = \frac{2}{3} (\tau + 37)^{3/2} \Big|_0^t \\ &= \frac{2}{3} (t + 37)^{3/2} - \frac{2}{3} (37)^{3/2} // \end{aligned}$$

2. (5 pts.) For the curve given in problem 1, find the length of the portion of the curve corresponding to the parameter range $2 \leq t \leq 6$.

$$L = \int_2^6 (\tau + 37)^{1/2} d\tau = s(6) - s(2)$$

$$= \frac{2}{3} (43)^{3/2} - \frac{2}{3} (37)^{3/2} - \frac{2}{3} (39)^{3/2} + \frac{2}{3} (37)^{3/2}$$

$$= \frac{2}{3} (43^{3/2} - 39^{3/2}) \approx 25.6 //$$