

MATH 213 - QUIZ 2 - 7 FEBRUARY 2008

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (3 pts.) Find parametric equations for the line containing the points (3, 2, -1) and (1, -1, 3).

$$\begin{array}{l}
 P = (3, 2, -1) \\
 Q = (1, -1, 3) \\
 \text{direction} \\
 \text{vector } \vec{v} = \vec{PQ} = \langle -2, -3, 4 \rangle
 \end{array}
 \quad \left| \begin{array}{l}
 \text{point: } (3, 2, -1) \\
 x = 3 - 2t \\
 y = 2 - 3t \\
 z = -1 + 4t
 \end{array} \right.
 \quad \left| \begin{array}{l}
 \text{or} \\
 \text{point: } (1, -1, 3) \\
 x = 1 - 2t \\
 y = -1 - 3t \\
 z = 3 + 4t
 \end{array} \right.$$

2. (2 pts.) Find the equation of the plane containing the point (3, -4, -1) normal to the vector $\mathbf{n} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. Put your final answer in the form $Ax + By + Cz = D$.

$$\begin{aligned}
 -2(x-3) + 2(y+4) - (z+1) &= 0 \\
 -2x + 6 + 2y + 8 - z - 1 &= 0 \\
 -2x + 2y - z &= -13 //
 \end{aligned}$$

3. (3 pts.) Find the area of the triangle determined by the points (1, 1, 1), (-1, 2, -2) and (0, 2, 1).

$$\begin{array}{l}
 P = (1, 1, 1) \\
 Q = (-1, 2, -2) \\
 R = (0, 2, 1)
 \end{array}
 \quad \begin{array}{l}
 \text{Area of } \Delta = \frac{1}{2} \text{ Area of parallelogram} \\
 \text{determined by } \vec{PQ} \text{ and } \vec{PR} \\
 = \frac{1}{2} |\vec{PQ} \times \vec{PR}| = \frac{1}{2} \sqrt{19} //
 \end{array}$$

$$\begin{array}{l}
 \vec{PQ} = \langle -2, 1, -3 \rangle \\
 \vec{PR} = \langle -1, 1, 0 \rangle
 \end{array}
 \quad \left| \vec{PQ} \times \vec{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & -3 \\ -1 & 1 & 0 \end{vmatrix} = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k} \right.$$

$$|\vec{PQ} \times \vec{PR}| = (9 + 9 + 1)^{1/2} = \sqrt{19} //$$

4. (2 pts.) Find the volume of the box determined by the three vectors $\mathbf{u} = \mathbf{i} - \mathbf{j}$, $\mathbf{v} = 3\mathbf{k}$, and $\mathbf{w} = 2\mathbf{j}$.

$$\begin{aligned}
 \text{Volume of box} &= |(\vec{v} \times \vec{w}) \cdot \vec{u}| \\
 \vec{v} \times \vec{w} &= (3\mathbf{k}) \times (2\mathbf{j}) = 6(\mathbf{k} \times \mathbf{j}) = -6\mathbf{i} \\
 (\vec{v} \times \vec{w}) \cdot \vec{u} &= (-6\mathbf{i}) \cdot (\mathbf{i} - \mathbf{j}) = -6 \\
 |(\vec{v} \times \vec{w}) \cdot \vec{u}| &= 6 //
 \end{aligned}$$

(Note: could also have taken $(\vec{u} \times \vec{v}) \cdot \vec{w}$ or $(\vec{u} \times \vec{w}) \cdot \vec{v}$ etc in any order and gotten same answer.