

MATH 213 - QUIZ 1 - 31 JANUARY 2008

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (1 pt. each) Let \mathbf{u} , \mathbf{v} and \mathbf{w} be vectors. Indicate whether each of the following quantities is a vector or a scalar (circle one).

- $\mathbf{u} \cdot \mathbf{v}$ SCALAR VECTOR
 $\text{proj}_{\mathbf{v}}(\mathbf{u})$ SCALAR VECTOR
 $\mathbf{u} \times \mathbf{v}$ SCALAR VECTOR
 $|\mathbf{u} \times \mathbf{v}|$ SCALAR VECTOR
 $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ SCALAR VECTOR

2. Let $\mathbf{u} = 5\mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = 6\mathbf{i} + 2\mathbf{k}$.

(a) (2 pts.) Find $\cos \theta$ where θ is the angle between \mathbf{u} and \mathbf{v} .

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{30}{2\sqrt{34} \cdot \sqrt{10}}$$

$$= \frac{15}{2\sqrt{85}} \approx .813 //$$

$$\vec{u} \cdot \vec{v} = (5\vec{i} + 3\vec{j}) \cdot (6\vec{i} + 2\vec{k})$$

$$= 30 + 0 + 0 = 30$$

$$|\vec{u}| = (25 + 9 + 0)^{1/2} = \sqrt{34}$$

$$|\vec{v}| = (36 + 0 + 4)^{1/2} = \sqrt{40} = 2\sqrt{10}$$

(b) (2 pts.) Find $\mathbf{u} \times \mathbf{v}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 3 & 0 \\ 6 & 0 & 2 \end{vmatrix} = 6\vec{i} - 10\vec{j} - 18\vec{k} //$$

$$\begin{matrix} \vec{i} \times \vec{j} \\ \vec{j} \times \vec{k} \end{matrix}$$

or

$$\vec{u} \times \vec{v} = (5\vec{i} + 3\vec{j}) \times (6\vec{i} + 2\vec{k}) = 30(\vec{i} \times \vec{i}) + 10(\vec{i} \times \vec{k}) + 18(\vec{j} \times \vec{i}) + 6(\vec{j} \times \vec{k})$$

$$= -10\vec{j} - 18\vec{k} + 6\vec{i} //$$

(c) (2 pts.) Find $\text{proj}_{\vec{v}}(\mathbf{u})$

$$\text{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v} = \frac{30}{40} \vec{v} = \frac{3}{4} (6\vec{i} + 2\vec{k}) = \frac{9}{2} \vec{i} + \frac{3}{2} \vec{k} //$$