

MATH 213 – 25 MARCH 2008 – EXAM 2

Answer each of the following questions. Show all work, as partial credit may be given.

1. (8 pts.) Show that the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 3y^2}{3x^2 + y^2}$ does not exist.
2. (8 pts. each) Consider the function $f(x, y, z) = x^3y + y^3z + xz^3$.
 - (a) Find ∇f .
 - (b) Find the directional derivative of f at the point $(1, 0, 2)$ and in the direction $\mathbf{v} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$.
 - (c) Find the maximum rate of change of f at the point $(1, 0, 2)$, and the direction in which f changes most rapidly at the point $(1, 0, 2)$.
3. (18 pts.) Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ when $w = xy + \ln(z)$, $x = v^2/u$, $y = u + v$, and $z = \cos(u)$.
4. (12 pts. each) Find f_x , f_y , f_{xx} , f_{yy} , and f_{xy} for each of the following functions.
 - (a) $f(x, y) = x \sin(xy)$
 - (b) $f(x, y) = \ln(xy^2)$
5. (12 pts.) Find f_{zx} , f_{yz} , f_{xy} , and f_{zyz} when $f(x, y, z) = \frac{yz^2}{x^3}$.
6. Consider the function $z = f(x, y)$ where $f(x, y) = xy + y$.
 - (a) (4 pts.) Sketch the level curve corresponding to $z = 1$.
 - (b) (6 pts.) Find $\nabla f(x, y)$, and evaluate it at the point $(1, 1/2)$.
 - (c) (4 pts.) Sketch the vector $\nabla f(1, 1/2)$ that you found in part (b) on the same set of axes as the sketch you made in part (a), putting the tail of the vector at the point $(1, 1/2)$. Does the gradient at that point appear to be normal to the level curve passing through the point?