MATH 114 – QUIZ 14 – 2 MAY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Find the first three terms in the Taylor series for the function $f(x) = x^{-1/2}$ with center point $a = 1$.

$f(1) = 1$

$f'(x) = -\frac{1}{2}x^{-3/2}$; $f'(1) = \frac{-1}{2}$

$f''(x) = \frac{3}{4}x^{-5/2}$; $f''(1) = \frac{3}{4}$

$$x^{-1/2} \sim f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 + \cdots$$

$$= 1 - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 + \cdots$$

//

2. (5 pts.) Given that $\frac{1}{1-t} = \sum_{k=0}^{\infty} t^k$ for all $|t| < 1$, find the Taylor series for the function $f(x) = \frac{x^2}{1+x}$ centered at $a = 0$.

$$\frac{x^2}{1+x} = x^2 \cdot \frac{1}{1-(-x)} = x^2 \sum_{k=0}^{\infty} (-x)^k = \sum_{k=0}^{\infty} (-1)^k x^{k+2}$$

valid for $|-x| < 1$, i.e. $|x| < 1$ //.

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1. (5 pts.) Find the first three terms in the Taylor series for the function $f(x) = x^{5/2}$ with center point $a = 1$.

$f(1) = 1$

$f'(x) = \frac{5}{2}x^{3/2}$; $f'(1) = \frac{5}{2}$

$f''(1) = \frac{15}{4}x^{1/2}$; $f''(1) = \frac{15}{4}$

$$x^{5/2} \sim f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 + \cdots$$

$$= 1 + \frac{5}{2}(x-1) + \frac{15}{8}(x-1)^2 + \cdots$$

//

2. (5 pts.) Given that $\frac{1}{1-t} = \sum_{k=0}^{\infty} t^k$ for all $|t| < 1$, find the Taylor series for the function $f(x) = \frac{x}{1-x^3}$ centered at $a = 0$.

$$\frac{x}{1-x^3} = x \cdot \frac{1}{1-(x^3)} = x \cdot \sum_{k=0}^{\infty} (x^3)^k = \sum_{k=0}^{\infty} x^{3k+1}$$

valid for $|x^3| < 1$, that is, $|x| < 1$. //

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$f(1) = 1$

$f'(x) = \frac{3}{2}x^{1/2}$; $f'(1) = \frac{3}{2}$

$f''(x) = \frac{3}{4}x^{-1/2}$; $f''(1) = \frac{3}{4}$

$$x^{3/2} \sim f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 + \cdots$$

$$= 1 + \frac{3}{2}(x-1) + \frac{3}{8}(x-1)^2 + \cdots \quad //$$

2. (5 pts.) Given that $\frac{1}{1-t} = \sum_{k=0}^{\infty} t^k$ for all $|t| < 1$, find the Taylor series for the function $f(x) = \frac{1}{1+x^2}$ centered at $a = 0$.

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{k=0}^{\infty} (-x^2)^k = \sum_{k=0}^{\infty} (-1)^k x^{2k}$$

valid for $|-x^2| < 1$, i.e., $|x| < 1$ //