MATH 114 – QUIZ 13 – 25 APRIL 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Find the quadratic approximating polynomial for the function $f(x) = \cos(x)$ at center point $a = 0$.

$f(0) = \cos(0) = 1$

$f'(x) = -\sin(x)$; $f'(0) = 0$

$f''(x) = -\cos(x)$; $f''(0) = -1$

$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$

$= 1 - \frac{1}{2}x^2$ //

2. (5 pts.) Identify the center a of the power series $\sum_{k=1}^{\infty} \left(\frac{x+3}{4}\right)^k$, and find its radius of convergence. $a = -3$

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x+3)^{n+1}}{4^{n+1}} \cdot \frac{4^n}{(x+3)^n}\right| = \frac{1}{4}|x+3| \to$

$\frac{1}{4}|x+3|$ as $n \to \infty$.

$\frac{1}{4}|x+3| < 1 \longrightarrow |x+3| < 4$

Radius of convergence $= 4$

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1. (5 pts.) Find the quadratic approximating polynomial for the function $f(x) = e^{-x^2}$ at center point $a = 0$.

$f'(x) = -2xe^{-x^2}$; $f'(0) = 0$

$f''(x) = 4x^2e^{-x^2} - 2e^{-x^2}$; $f''(0) = -2$

$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 = 1 - x^2$ //

2. (5 pts.) Identify the center a of the power series $\sum_{k=0}^{\infty} \frac{k^2}{2^k}(x-1)^k$, and find its radius of convergence.

$a = 1$

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)^2(x-1)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n^2(x-1)^n}\right|$

$= \left(\frac{n+1}{n}\right)^2 \cdot \frac{1}{2}|x-1| \to \frac{1}{2}|x-1|$ as $n \to \infty$

$\frac{1}{2}|x-1| < 1 \implies |x-1| < 2$ $\therefore$ radius of convergence $= 2$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Find the quadratic approximating polynomial for the function $f(x) = \dfrac{1}{1-x}$ at center point $a = 0$.

$f(0) = 1$

$f'(x) = \dfrac{1}{(1-x)^2}$; $f'(0) = 1$

$f''(x) = \dfrac{2}{(1-x)^3}$; $f''(0) = 2$

$$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

$$= 1 + x + x^2 \;//$$

2. (5 pts.) Identify the center a of the power series $\sum_{k=1}^{\infty}(-1)^k \dfrac{2^k}{k^2}(x-4)^k$, and find its radius of convergence. $a = 4$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-1)^{n+1}2^{n+1}(x-4)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n 2^n (x-4)^n}\right|$$

$$= \left|\left(\frac{n}{n+1}\right)^2 \cdot 2\,(x-4)\right| = \left(\frac{n}{n+1}\right)^2 \cdot 2\,|x-4|$$

$\longrightarrow 2|x-4|$ as $n \to \infty$.

$2|x-4| < 1 \longrightarrow |x-4| < \frac{1}{2}$

Radius of convergence $R = \frac{1}{2}$