MATH 114 – QUIZ 12 – 18 APRIL 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Use the ratio test or root test to determine whether or not the series $\sum_{k=1}^{\infty} k^3 e^{-k}$ converges.

Ratio Test: $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)^3 e^{-n}}{n^3 e^{-(n+1)}}\right| = \left(\frac{n+1}{n}\right)^3 \cdot e^{-1}$

$\to e^{-1}$ as $n \to \infty$. Since $e^{-1} < 1$ series converges

Root test: $|a_n|^{1/n} = |n^3 e^{-n}|^{1/n} = n^{\frac{3}{n}} (e^{-n})^{1/n}$

$= (n^{1/n})^3 e^{-1} \longrightarrow e^{-1}$ as $n \to \infty$. Since $e^{-1} < 1$

series converges.

2. (5 pts.) Use the alternating series test to determine whether or not the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{100+k}$ converges or diverges. (For an extra bonus point: does it converge absolutely? Why or why not? I need a reason, but not necessarily a proof.)

(1) $\frac{1}{100+k} > 0$ all k

(2) $\frac{1}{100+k}$ is decreasing

(3) $\lim_{k\to\infty} \frac{1}{100+k} = 0$

$\therefore$ By AST,

$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{100+k}$ converges

$\sum_{1}^{\infty} \left|(-1)^{k+1} \frac{1}{100+k}\right|$

$= \sum_{1}^{\infty} \frac{1}{100+k}$ diverges

since $\frac{1}{100+k} \sim \frac{1}{k}$ and

$\sum \frac{1}{k}$ diverges.

Series does not converge absolutely

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1. (5 pts.) Use the ratio test or root test to determine whether or not the series $\sum_{k=1}^{\infty} \frac{k^2}{4^k}$ converges.

Ratio test: $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)^2}{4^{n+1}} \cdot \frac{4^n}{n^2}\right| = \left(\frac{n+1}{n}\right)^2 \cdot \frac{1}{4} \to \frac{1}{4}$

as $n \to \infty$. Since $\frac{1}{4} < 1$ the series converges.

Root test: $|a_n|^{1/n} = \left(\frac{n^2}{4^n}\right)^{1/n} = n^{\frac{2}{n}} \cdot \frac{1}{(4^n)^{1/n}} = (n^{1/n})^2 \cdot \frac{1}{4}$

$\longrightarrow \frac{1}{4}$ as $n \to \infty$. Since $\frac{1}{4} < 1$ the series converges.

2. (5 pts.) Use the alternating series test to determine whether or not the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{\sqrt{k+1}}$ converges or diverges. (For an extra bonus point: does it converge absolutely? Why or why not? I need a reason, but not necessarily a proof.)

(1) $\frac{1}{\sqrt{k+1}} > 0$ all k

(2) $\frac{1}{\sqrt{k+1}}$ is decreasing

(3) $\lim_{k \to \infty} \frac{1}{\sqrt{k+1}} = 0$

$\therefore$ By A.S.T. $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{\sqrt{k+1}}$ converges. //

$\sum_{k=1}^{\infty} \left|(-1)^{k+1} \frac{1}{\sqrt{k+1}}\right| = \sum_{k=1}^{\infty} \frac{1}{\sqrt{k+1}}$ diverges (it is like a p-series with $p = \frac{1}{2} < 1$)

Series does not converge absolutely

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1. (5 pts.) Use the ratio test or root test to determine whether or not the series $\sum_{k=1}^{\infty} \frac{4^k}{k!}$ converges.

Ratio test: $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n}\right| = \frac{4}{n+1} \to 0$ as $n \to \infty$.

Since $0<1$ the series converges.

Note: Root test is difficult to apply.

2. (5 pts.) Use the alternating series test to determine whether or not the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k^4+1}$ converges or diverges. (For an extra bonus point: does it converge absolutely? Why or why not? I need a reason, but not necessarily a proof.)

(1) $\frac{k^2}{k^4+1} > 0$ all k

(2) $\frac{k^2}{k^4+1}$ is decreasing

Note: $\frac{d}{dx}\left(\frac{x^2}{x^4+1}\right) = \frac{2x(x^4+1)-4x^5}{(x^4+1)^2}$

$= \frac{-2x^5+2x}{(x^4+1)^2} = \frac{2x}{(x^4+1)^2}(1-x^4) < 0$ if $x>1$

so $\frac{x^2}{x^4+1}$ is decreasing if $x>1$

(3) $\lim_{k\to\infty} \frac{k^2}{k^4+1} = 0$.

By AST

$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k^4+1}$ converges

$\sum_{k=1}^{\infty}\left|(-1)^{k+1}\frac{k^2}{k^4+1}\right| = \sum_{k=1}^{\infty}\frac{k^2}{k^4+1}$

converges since

$\frac{k^2}{k^4+1} \sim \frac{1}{k^2}$ and $\sum \frac{1}{k^2}$ converges

$\therefore$ series converges absolutely