MATH 114 – QUIZ 11 – 11 APRIL 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Use the Divergence Test or the Integral Test to determine if the series $\sum_{k=1}^{\infty} \sqrt{\frac{k}{k+1}}$ converges or diverges. You may assume that all conditions for the Integral Test are satisfied.

$$\lim_{k\to\infty} \sqrt{\frac{k}{k+1}} = \sqrt{\lim_{k\to\infty} \frac{k}{k+1}} = \sqrt{1} = 1 \neq 0.$$

Series diverges by Divergence Test

2. (5 pts.) Use the Divergence Test or the Integral Test to determine if the series $\sum_{k=1}^{\infty} \frac{1}{(k+1)^{3/2}}$ converges or diverges. You may assume that all conditions for the Integral Test are satisfied.

$$\int_1^{\infty} \frac{1}{(x+1)^{3/2}}\,dx = \lim_{b\to\infty} \int_1^b (x+1)^{-3/2}\,dx$$

$$= \lim_{b\to\infty} -2(x+1)^{-1/2}\Big|_1^b = \lim_{b\to\infty} \frac{-2}{(b+1)^{1/2}} + \frac{2}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}}.$$ Series converges by integral test.

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1. (5 pts.) Use the Divergence Test or the Integral Test to determine if the series $\sum_{k=1}^{\infty} \frac{k}{\sqrt{k^2+1}}$ converges or diverges. You may assume that all conditions for the Integral Test are satisfied.

$$\lim_{k \to \infty} \frac{k}{\sqrt{k^2+1}} = \lim_{k \to \infty} \sqrt{\frac{k^2}{k^2+1}} = \sqrt{\lim_{k \to \infty} \frac{k^2}{k^2+1}}$$

$= \sqrt{1} = 1 \neq 0$. Series diverges by Divergence Test.

2. (5 pts.) Use the Divergence Test or the Integral Test to determine if the series $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$ converges or diverges. You may assume that all conditions for the Integral Test are satisfied.

$$\int_2^{\infty} \frac{1}{x(\ln x)^3}\,dx = \lim_{b \to \infty} \int_2^b \frac{1}{x(\ln x)^3}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$x = 2 \to u = \ln 2$

$x = b \to u = \ln b$.

$$= \lim_{b \to \infty} \int_{\ln 2}^{\ln b} u^{-3}\,du = \lim_{b \to \infty} -\frac{1}{2} u^{-2} \Big|_{\ln 2}^{\ln b}$$

$$= \lim_{b \to \infty} -\frac{1}{2(\ln b)^2} + \frac{1}{2(\ln 2)^2} = \frac{1}{2(\ln 2)^2}$$

Series converges by integral test.

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1. (5 pts.) Use the Divergence Test or the Integral Test to determine if the series $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$ converges or diverges. You may assume that all conditions for the Integral Test are satisfied.

$$\int_1^{\infty} \frac{x}{x^2+1}\,dx = \frac{1}{2} \lim_{b\to\infty} \int_1^b \frac{2x}{x^2+1}\,dx$$

$u = x^2+1$
$du = 2x\,dx$
$x=1 \rightarrow u=2$
$x=b \rightarrow u=b^2+1$

$$= \frac{1}{2} \lim_{b\to\infty} \int_2^{b^2+1} \frac{1}{u}\,du = \frac{1}{2} \lim_{b\to\infty} \left(\ln(b^2+1) - \ln 2\right) = \infty$$

Series diverges by integral test.

2. (5 pts.) Use the Divergence Test or the Integral Test to determine if the series $\sum_{k=2}^{\infty} \frac{k^3}{k^3+1}$ converges or diverges. You may assume that all conditions for the Integral Test are satisfied.

$$\lim_{k\to\infty} \frac{k^3}{k^3+1} = 1 \neq 0.$$

Series diverges by Divergence Test.