MATH 114 – QUIZ 10 – 4 APRIL 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Evaluate the series $\sum_{k=2}^{\infty} 2 \cdot 3^{-k}$.

$$\sum_{k=2}^{\infty} 2 \cdot 3^{-k} = 2 \sum_{k=2}^{\infty} 3^{-k} = 2 \cdot 3^{-2} \sum_{k=0}^{\infty} 3^{-k} = \frac{2}{9} \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^{k}$$

$$= \frac{2}{9} \cdot \frac{1}{1-\frac{1}{3}} = \frac{2}{9} \cdot \frac{3}{2} = \frac{1}{3} //$$

2. (4 pts.) Evaluate the telescoping series $\sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+2}}\right)$.

$$S_n = \sum_{k=1}^{n} \left(\frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+2}}\right)$$

$$= \left(\frac{1}{\sqrt{2}} - \cancel{\frac{1}{\sqrt{3}}}\right) + \left(\cancel{\frac{1}{\sqrt{3}}} - \cancel{\frac{1}{\sqrt{4}}}\right) + \cancel{\cdots} + \left(\cancel{\frac{1}{\sqrt{n+1}}} - \frac{1}{\sqrt{n+2}}\right)$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{n+2}} \to \frac{1}{\sqrt{2}} \text{ as } n \to \infty.$$

$$\therefore \sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{k+1}} - \frac{1}{\sqrt{k+2}}\right) = \frac{1}{\sqrt{2}} //$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Evaluate the series $\sum_{k=0}^{\infty} 3^{-2k}$.

$$\sum_{k=0}^{\infty} 3^{-2k} = \sum_{k=0}^{\infty} (3^{-2})^k = \sum_{k=0}^{\infty} \left(\frac{1}{9}\right)^k$$

$$= \frac{1}{1-\frac{1}{9}} = \frac{9}{8} //$$

2. (4 pts.) Evaluate the telescoping series $\sum_{k=1}^{\infty} \left(\frac{1}{k+2} - \frac{1}{k+3}\right)$.

$$S_n = \sum_{k=1}^{n} \left(\frac{1}{k+2} - \frac{1}{k+3}\right)$$

$$= \left(\frac{1}{3} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \cancel{\cdots} + \left(\cancel{\frac{1}{n+2}} - \frac{1}{n+3}\right)$$

$$= \frac{1}{3} - \frac{1}{n+3} \to \frac{1}{3} \quad \text{as } n \to \infty$$

$$\therefore \sum_{k=1}^{\infty} \left(\frac{1}{k+2} - \frac{1}{k+3}\right) = \frac{1}{3} //$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Evaluate the series $\sum_{k=2}^{\infty} \frac{4^k}{5^k}$.

$$\sum_{k=2}^{\infty} \frac{4^k}{5^k} = \sum_{k=2}^{\infty} \left(\frac{4}{5}\right)^k = \left(\frac{4}{5}\right)^2 \sum_{k=0}^{\infty} \left(\frac{4}{5}\right)^k$$

$$= \frac{16}{25} \cdot \frac{1}{1-\frac{4}{5}} = \frac{16}{5} //$$

2. (4 pts.) Evaluate the telescoping series $\sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right)$.

$$S_n = \sum_{k=2}^{n} \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

$$= \left(\frac{1}{2} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) \cancel{+} \cdots \cancel{+} \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$$

$$= \frac{1}{2} - \frac{1}{n+1} \to \frac{1}{2} \text{ as } n \to \infty$$

$$\therefore \sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right) = \frac{1}{2} //$$