MATH 114 – QUIZ 8 – 21 MARCH 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Consider the integral $\int_1^3 \frac{2}{t^3}\,dt$. Estimate this integral using the trapezoid rule and Simpson's rule for $n = 2$ (rounded to two decimal places). Given that $\int_1^3 \frac{2}{t^3}\,dt = \frac{8}{9}$, find the percentage error in each case (rounded to the nearest percent).

$$T(2) = \frac{\Delta x}{2}\left(f(1)+2f(2)+f(3)\right) = \frac{1}{2}\left(2+\frac{1}{2}+\frac{2}{27}\right)$$

$$\approx 1.29$$

$$S(2) = \frac{\Delta x}{3}\left(f(1)+4f(2)+f(3)\right) = \frac{1}{3}\left(2+1+\frac{2}{27}\right)$$

$$\approx 1.02$$

$$E_T = \frac{|1.29 - \frac{8}{9}|}{|\frac{8}{9}|} \approx 45\% \qquad E_S = \frac{|1.02 - \frac{8}{9}|}{|\frac{8}{9}|} \approx 15\%$$

2. (4 pts.) Find the value of $\int_1^\infty \frac{1}{t^2+1}\,dt$. (Hint: $\lim_{t\to\infty} \tan^{-1}(t) = \pi/2$.)

$$\int_1^\infty \frac{1}{t^2+1}\,dt = \lim_{b\to\infty}\int_1^b \frac{1}{t^2+1}\,dt = \lim_{b\to\infty} \tan^{-1}(t)\Big|_1^b$$

$$= \lim_{b\to\infty} \tan^{-1}(b) - \tan^{-1}(1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Consider the integral $\int_1^3 t^4\,dt$. Estimate this integral using the trapezoid rule and Simpson's rule for $n = 2$ (rounded to two decimal places). Given that $\int_1^3 t^4\,dt = 48.6$, find the percentage error in each case (rounded to the nearest percent).

$$T(2) = \frac{\Delta x}{2}(f(1)+2f(2)+f(3)) = \frac{1}{2}(1+32+81) = 57.0$$

$$S(2) = \frac{\Delta x}{3}(f(1)+4f(2)+f(3)) = \frac{1}{3}(1+64+81) = \frac{146}{3} \approx 48.67$$

$$E_T = \frac{|57-48.6|}{|48.6|} \approx 17\% \qquad E_S = \frac{|48.67-48.6|}{|48.6|} \approx 0.13\%$$

2. (4 pts.) Find the value of $\int_1^\infty \frac{1}{t^3}\,dt$.

$$= \lim_{b\to\infty} \int_1^b t^{-3}\,dt = \lim_{b\to\infty} -\frac{1}{2}t^{-2}\Big|_1^b = \lim_{b\to\infty} -\frac{1}{2}b^{-2} + \frac{1}{2}$$

$$= \frac{1}{2}$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Consider the integral $\int_0^2 t^4 + t\,dt$. Estimate this integral using the trapezoid rule and Simpson's rule for $n = 2$ (rounded to two decimal places). Given that $\int_0^2 t^4 + t\,dt = 8.4$, find the percentage error in each case (rounded to the nearest percent).

$$T(2) = \frac{\Delta x}{2}(f(0) + 2f(1) + f(2)) = \frac{1}{2}(0 + 4 + 18) = 11$$

$$S(2) = \frac{\Delta x}{3}(f(0) + 4f(1) + f(2)) = \frac{1}{3}(0 + 8 + 18) = \frac{26}{3} \approx 8.67$$

$$E_T = \frac{|11 - 8.4|}{|8.4|} \approx 31\% \qquad E_S = \frac{|8.67 - 8.4|}{8.4} = 3\%$$

2. (4 pts.) Find the value of $\int_1^\infty e^{-2t}\,dt$.

$$\int_1^\infty e^{-2t}\,dt = \lim_{b\to\infty} \int_1^b e^{-2t}\,dt = \lim_{b\to\infty} -\frac{1}{2}e^{-2t}\Big|_1^b$$

$$= \lim_{b\to\infty} -\frac{1}{2}e^{-2b} + \frac{1}{2}e^{-2} = \frac{1}{2}e^{-2}$$