MATH 114 – QUIZ 6 – 28 FEBRUARY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (10 pts.) Evaluate the integral $\int \frac{x^2}{(1+x^2)^{1/2}}\,dx$. (Hint: $1+\tan^2\theta = \sec^2\theta$; and $\frac{d}{dx}(\tan\theta) = \sec^2\theta$).

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$(1+x^2)^{1/2} = (1+\tan^2\theta)^{1/2} = (\sec^2\theta)^{1/2} = \sec\theta$

$$= \int \frac{\tan^2\theta}{\sec\theta}\sec^2\theta\,d\theta = \int \tan^2\theta\sec\theta\,d\theta$$

Note: This problem was incorrectly written and the resulting integral is very hard. My apologies. Grading for this quiz will be adjusted.
Here is a way to do this problem:

$$\int \tan^2\theta\sec\theta\,d\theta = \int \frac{\sin^2\theta}{\cos^3\theta}\,d\theta = \int \frac{\sin^2\theta}{\cos^4\theta}\cos\theta\,d\theta$$

$$= \int \frac{\sin^2\theta}{(1-\sin^2\theta)^2}\cos\theta\,d\theta \underset{\substack{u=\sin\theta\\ du=\cos\theta\,d\theta}}{=} \int \frac{u^2}{(1-u^2)^2}\,du$$ ← Partial fractions

$$\frac{u^2}{(1-u^2)^2} = \frac{u^2}{(1-u)^2(1+u)^2} = \frac{A}{1-u} + \frac{B}{(1-u)^2} + \frac{C}{1+u} + \frac{D}{(1+u)^2}$$

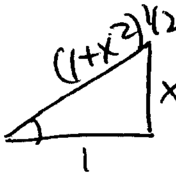
$$= \frac{1}{4}\left[\frac{-1}{1-u} + \frac{1}{(1-u)^2} - \frac{1}{1+u} + \frac{1}{(1+u)^2}\right]$$ ← This after lots of tedious calculating

whew!

$$= \frac{1}{4}\left[\ln|u-1| - \ln|u+1| + \frac{1}{1-u} - \frac{1}{1+u}\right] + C$$

$\theta = \tan^{-1}(x)$

$u = \sin(\tan^{-1}x)$



$u = \frac{x}{(1+x^2)^{1/2}}$

$$= \frac{1}{4}\left[\ln\left|\frac{\frac{x}{(1+x^2)^{1/2}} - 1}{\frac{x}{(1+x^2)^{1/2}} + 1}\right| + \frac{1}{\left(1 - \frac{x}{(1+x^2)^{1/2}}\right)} + \frac{1}{1 + \frac{x}{(1+x^2)^{1/2}}}\right]$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (10 pts.) Evaluate the integral $\int \frac{dx}{(1+x^2)^{3/2}}$. (Hint: $1+\tan^2\theta = \sec^2\theta$; and $\sec\theta = 1/\cos\theta$.)

$x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

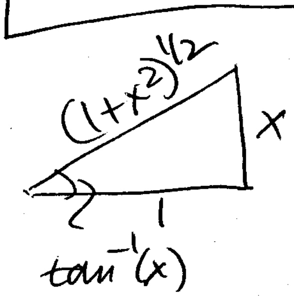
$(1+x^2)^{3/2} = (1+\tan^2\theta)^{3/2} = (\sec^2\theta)^{3/2} = \sec^3\theta$

$$\int \frac{dx}{(1+x^2)^{3/2}} = \int \frac{\sec^2\theta}{\sec^3\theta}\, d\theta = \int \frac{1}{\sec\theta}\, d\theta$$

$$= \int \cos\theta \, d\theta = \sin\theta + C$$

$\boxed{\theta = \tan^{-1}(x)}$

$$= \sin(\tan^{-1} x) + C$$



$$= \frac{x}{(1+x^2)^{1/2}} + C \quad //$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (10 pts.) Evaluate the integral $\int \frac{dx}{x^2(x^2-1)^{1/2}}$. (Hint: $\tan^2\theta = \sec^2\theta - 1$; and $\sec\theta = 1/\cos\theta$.)

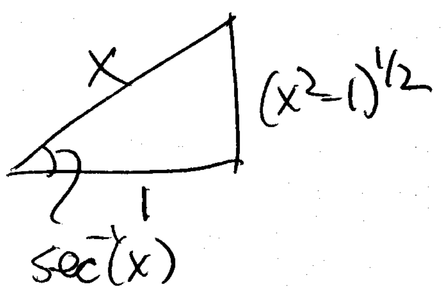
$$x = \sec\theta \qquad \theta = \sec^{-1}(x)$$

$$dx = \sec\theta\tan\theta\, d\theta$$

$$(x^2-1)^{1/2} = (\sec^2\theta - 1)^{1/2} = (\tan^2\theta)^{1/2} = \tan\theta$$

$$\int \frac{\sec\theta\tan\theta}{\sec^2\theta\tan\theta}\, d\theta = \int \frac{1}{\sec\theta}\, d\theta = \int \cos\theta\, d\theta$$

$$= \sin\theta + c = \sin(\sec^{-1}(x)) + c$$



$$= \frac{(x^2-1)^{1/2}}{x} + c \quad //$$