MATH 114 – QUIZ 5 – 21 FEBRUARY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Compute the integral $\int x^2 \sin(x)\,dx$ by integration by parts.

$u = x^2 \quad dv = \sin(x)\,dx$

$du = 2x\,dx \quad v = -\cos(x)$

$$\int x^2 \sin(x)\,dx = -x^2\cos(x) + 2\int x\cos(x)\,dx \quad \begin{cases} u = x & dv = \cos(x)\,dx \\ du = dx & v = \sin(x) \end{cases}$$

$$= -x^2\cos(x) + 2\left[x\sin(x) - \int \sin(x)\,dx\right]$$

$$= -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C$$

2. (5 pts.) Compute the definite integral $\int_0^{\pi/2} \cos^3(x)\,dx$.

$$\int_0^{\pi/2} \cos^3(x)\,dx = \int_0^{\pi/2} \cos^2(x)\cos(x)\,dx = \int_0^{\pi/3} \left(1 - \sin^2(x)\right)\cos(x)\,dx$$

$u = \sin(x)$

$du = \cos(x)\,dx$

$x = 0 \quad u = 0$

$x = \pi/3 \quad u = 1$

$$= \int_0^1 (1 - u^2)\,du = u - \frac{1}{3}u^3\Big|_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$$

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Compute the integral $\int x^2 e^{-x}\,dx$ by integration by parts.

$u = x^2 \quad dv = e^{-x}dx$

$du = 2x\,dx \quad v = -e^{-x}$

$= -x^2e^{-x} + 2\int xe^{-x}dx$ → $u = x \quad dv = e^{-x}$, $du = dx \quad v = -e^{-x}$

$= -x^2e^{-x} + 2\left[-xe^{-x} + \int e^{-x}dx\right]$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$ //

2. (5 pts.) Compute the definite integral $\int_0^{\pi/2} \sin^3(x)\,dx$.

$\int_0^{\pi/2} \sin^3(x)\,dx = \int_0^{\pi/2} \sin^2(x)\sin(x)\,dx = \int_0^{\pi/2} (1-\cos^2(x))\sin(x)\,dx$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$x = 0 \quad u = 1$

$x = \frac{\pi}{2} \quad u = 0$

$= -\int_1^0 (1-u^2)\,du = \int_0^1 (1-u^2)\,du$

$= u - \frac{1}{3}u^3 \Big|_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$ //

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Compute the integral $\int x^2 \ln(x)\,dx$ by integration by parts. (Hint: When writing the integral as $\int u\,dv$ try $u = \ln(x)$.)

$u = \ln x \quad dv = x^2 dx$

$du = \frac{1}{x} dx \quad v = \frac{1}{3}x^3$

$= \frac{1}{3}x^3 \ln(x) - \frac{1}{3}\int x^2 dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$ //

2. (5 pts.) Compute the definite integral $\int_0^{\pi/4} \tan^2(x)\,dx$. (Hint: $\tan^2(x) = \sec^2(x) - 1$.)

$\int_0^{\pi/4} (\sec^2(x) - 1)\,dx = \tan(x) - x \Big|_0^{\pi/4} = \tan\frac{\pi}{4} - \frac{\pi}{4} - \tan(0) + 0$

$= 1 - \frac{\pi}{4}$ //