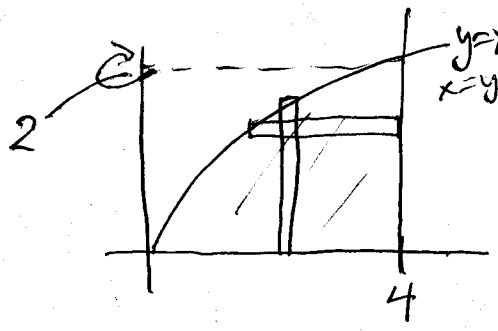
MATH 114 – QUIZ 4 – 14 FEBRUARY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^{1/2}$, $y = 0$, and $x = 4$ around the $y$–axis. Use any method you like.




① washers.

$$V = \int_0^2 \pi\left(4^2 - (y^2)^2\right) dy$$

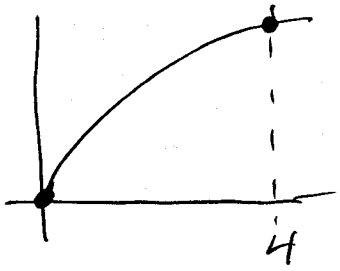
$$= \int_0^2 \pi\left(16 - y^4\right) dy = \pi\left(16y - \tfrac{1}{5}y^5 \Big|_0^2\right)$$

$$= \pi\left(32 - \tfrac{32}{5}\right) = \tfrac{128}{5}\pi$$

② Shells: $V = \int_0^4 2\pi(x)(x^{1/2})dx = \int_0^4 2\pi x^{3/2} dx = 2\pi \tfrac{2}{5} x^{5/2}\Big|_0^4$

$$= 2\pi \cdot \tfrac{2}{5}(4)^{5/2} = 2\pi \cdot \tfrac{2}{5} \cdot 32 = \tfrac{128\pi}{5}$$

2. (4 pts.) Write down a definite integral that gives the length of the curve $y = x^{1/2}$ for $x$ between 0 and 4. You do not have to evaluate the integral.




$$dL = \sqrt{1 + (y')^2}\, dx$$

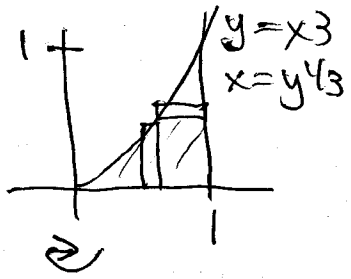
$$= \sqrt{1 + \left(\tfrac{1}{2}x^{-1/2}\right)^2} dx$$

$$L = \int_0^4 \left(1 + \tfrac{1}{4x}\right)^{1/2} dx$$

MATH 114 – QUIZ 4 – 14 FEBRUARY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^3$, $y = 0$, and $x = 1$ around the $y$–axis. Use any method you like.




① Washers:

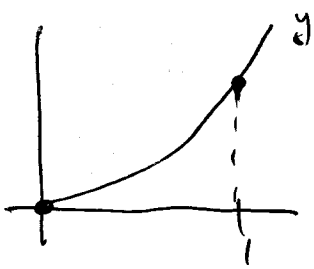
$$\int_0^1 \pi(1^2 - (y^{1/3})^2)\,dy = \pi\int_0^1 (1-y^{2/3})\,dy$$

$$= \pi\left(y - \frac{3}{5}y^{5/3}\Big|_0^1\right) = \pi\left(1-\frac{3}{5}\right) = \frac{2\pi}{5}$$

② Shells: $\int_0^1 2\pi(x)(x^3)\,dx = 2\pi\int_0^1 x^4\,dx$

$$= \frac{2\pi}{5}x^5\Big|_0^1 = \frac{2\pi}{5}$$

2. (4 pts.) Write down a definite integral that gives the length of the curve $y = x^3$ between $x = 0$ and $x = 1$. You do not have to evaluate the integral.




$$dL = \sqrt{1+(y')^2}\,dx$$

$$= \sqrt{1+(3x^2)^2}\,dx$$

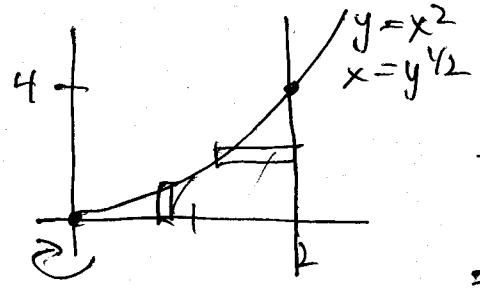
$$= \sqrt{1+9x^4}\,dx$$

$$L = \int_0^1 \sqrt{1+9x^4}\,dx$$

MATH 114 – QUIZ 4 – 14 FEBRUARY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^2$, $y = 0$, and $x = 2$ around the $y$–axis. Use any method you like.




① Washers: $\int_0^4 \pi\left(2^2 - (y^{1/2})^2\right)dy$

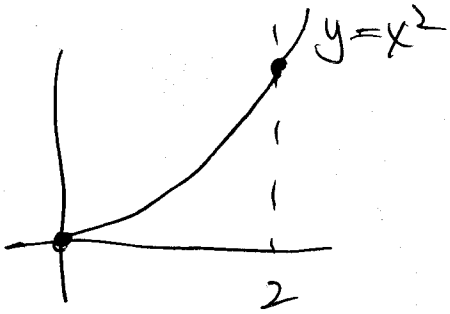
$= \int_0^4 \pi(4-y)dy = \pi\left(4y - \frac{1}{2}y^2\Big|_0^4\right)$

$= \pi(16-8) = 8\pi$ //

② Shells: $\int_0^2 2\pi(x)(x^2)dx = 2\pi\int_0^2 x^3 dx = 2\pi \cdot \frac{1}{4}x^4\Big|_0^2$

$= 2\pi\left(\frac{16}{4}\right) = 8\pi$ //

2. (4 pts.) Write down a definite integral that gives the length of the curve $y = x^2$ for $x$ between 0 and 2. You do not have to evaluate the integral.




$dL = \sqrt{1+(y')^2}\,dx$

$= \sqrt{1+(2x)^2}\,dx$

$= \sqrt{1+4x^2}\,dx$

$L = \int_0^2 (1+4(x^2))^{1/2}\,dx$ //