MATH 114 – QUIZ 2 – 1 FEBRUARY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) The population of a colony of prairie dogs grows at a rate given by $P'(t) = 20 - \frac{1}{5}t^2$ prairie dogs per month, for $0 \le t \le 10$ months.

(a) How much did the population change between $t = 3$ and $t = 6$ months.

$$P(6) - P(3) = \int_3^6 20 - \frac{1}{5}t^2\,dt = 20t - \frac{1}{15}t^3 \Big|_3^6$$

$$= 20(6) - \frac{1}{15}(6^3) - 20(3) + \frac{1}{15}(3)^3$$

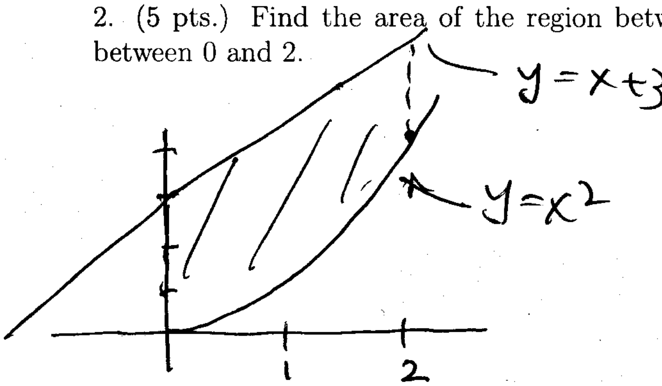
$$= 120 - \frac{72}{5} - 60 + \frac{9}{5} = \frac{237}{5} \approx 47 \text{ dogs.} //$$

(b) If $P(0) = 100$, find a formula for $P(t)$.

$$P(t) = \int_0^t P(x)\,dx + P(0) = \int_0^t 20 - \frac{1}{5}x^2\,dx + 100$$

$$= 20x - \frac{1}{15}x^3 \Big|_0^t + 100 = 20t - \frac{1}{15}t^3 + 100 //$$

2. (5 pts.) Find the area of the region between the curves $y = x + 3$ and $y = x^2$ for x between 0 and 2.



$$A = \int_0^2 x + 3 - x^2\,dx$$

$$= \frac{1}{2}x^2 + 3x - \frac{1}{3}x^3 \Big|_0^2$$

$$= 2 + 6 - \frac{8}{3} - 0 = \frac{16}{3} //$$

MATH 114 – QUIZ 2 – 1 FEBRUARY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Water is pumped into a cistern at a rate of $Q'(t) = 3\sqrt{t}$ gallons per minute for $0 \leq t \leq 100$ minutes.

(a) How much water is pumped in during the first hour (that is between $t = 0$ and $t = 60$).

$$Q(60) - Q(0) = \int_0^{60} 3t^{1/2}\,dt = 2t^{3/2}\Big|_0^{60}$$

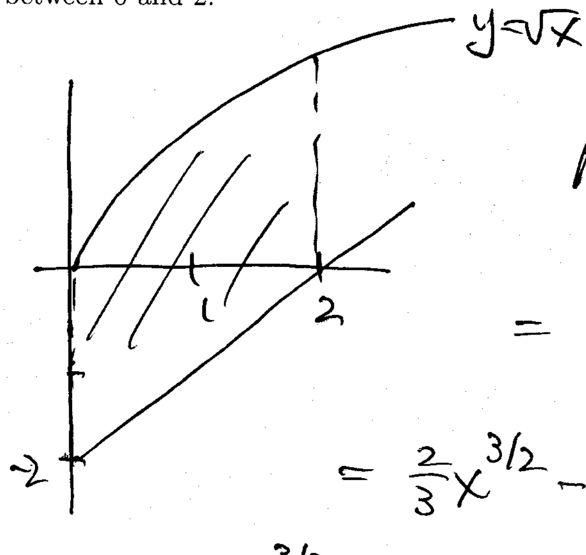
$$= 2(60)^{3/2} - 0 = 2(60)^{3/2} \text{ gal } (\approx 930 \text{ gal})$$

(b) If initially the cistern had $Q(0) = 100$ gallons of water in it, find a formula for $Q(t)$.

$$Q(t) = \int_0^t Q'(x)\,dx + Q(0) = \int_0^t 3x^{1/2}\,dx + 100$$

$$= 2x^{3/2}\Big|_0^t + 100 = 2t^{3/2} + 100 \; //$$

2. (5 pts.) Find the area of the region between the curves $y = \sqrt{x}$ and $y = x - 2$ for x between 0 and 2.



$$A = \int_0^2 x^{1/2} - (x-2)\,dx$$

$$= \int_0^2 x^{1/2} - x + 2\,dx$$

$$= \frac{2}{3}x^{3/2} - \frac{1}{2}x^2 + 2x \Big|_0^2$$

$$= \frac{2}{3}(2)^{3/2} - 2 + 4 - 0 = 2 + \frac{2}{3}(2)^{3/2} \; //.$$

MATH 114 – QUIZ 2 – 1 FEBRUARY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Suppose that a population of bacteria grows at a rate of $P(t) = 10\,e^{2t}$ bacteria per day for $0 \le t \le 10$ days.

(a) By how much does the population change during the first week (that is between $t = 0$ and $t = 7$).

$$P(7) - P(0) = \int_0^7 10\,e^{2t}\,dt = 5e^{2t}\Big|_0^7$$

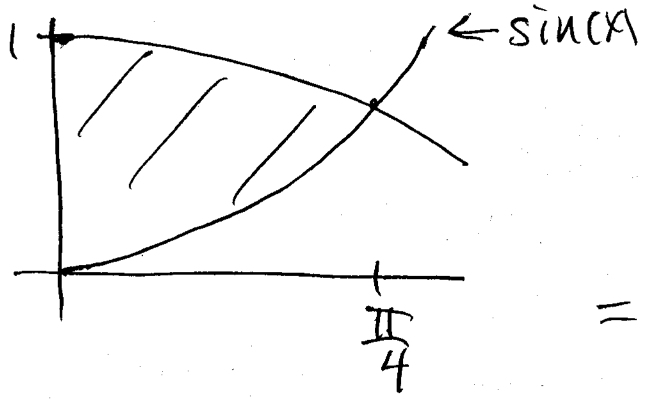
$$= 5e^{14} - 5 = 5(e^{14} - 1)$$

(b) If $P(0) = 100$, find a formula for $P(t)$.

$$P(t) = \int_0^t 10e^{2x}\,dx + P(0) = 5e^{2x}\Big|_0^t + 100$$

$$= 5e^{2t} - 5 + 100 = 5e^{2t} + 95$$

2. (5 pts.) Find the area of the region between the curves $y = \sin(x)$ and $y = \cos(x)$ for x between 0 and $\pi/4$.



$$A = \int_0^{\pi/4} \cos x - \sin x\,dx$$

$$= \sin(x) + \cos(x)\Big|_0^{\frac{\pi}{4}}$$

$$= \sin\left(\tfrac{\pi}{4}\right) + \cos\left(\tfrac{\pi}{4}\right) - \sin(0) - \cos(0)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1$$

$$= \sqrt{2} - 1$$