MATH 114 – QUIZ 1 – 24 JANUARY 2013

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

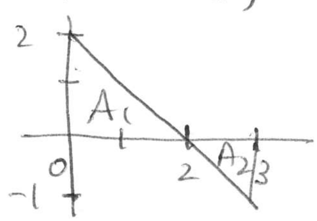
1. (6 pts.) Find the total displacement over the time interval $0 \leq t \leq 3$ of an object whose velocity function is given by $v(t) = 2 - t$. Find also the total distance travelled over that time interval. (Hint: You don't have to use calculus to solve this problem.)

Displacement $= \int_0^3 (2-t)\,dt = 2t - \frac{1}{2}t^2 \Big|_0^3 = 6 - \frac{9}{2} - 0 = \frac{3}{2}$ //

Distance travelled $= \int_0^3 |2-t|\,dt$

Graphically, $= A_1 + A_2 = \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1)$

$= \frac{5}{2}$ //



or, $\int_0^3 |2-t|\,dt = \int_0^2 (2-t)\,dt + \int_2^3 (t-2)\,dt$

$= \left(2t - \frac{1}{2}t^2 \Big|_0^2\right) + \left(\frac{1}{2}t^2 - 2t \Big|_2^3\right) = (4-2) + \left(\frac{9}{2} - 6 - 2 + 4\right)$

$= 2 + \frac{9}{2} - 4 = \frac{5}{2}$

2. (4 pts.) Evaluate the following definite integral using substitution. $\int_0^2 (2x+1)^{1/3}\,dx$

$u = 2x + 1$

$du = 2\,dx$

$x = 0 \rightarrow u = 1$

$x = 2 \rightarrow u = 5$

$= \int_1^5 u^{1/3} \cdot \frac{1}{2}\,du$

$= \frac{1}{2} \int_1^5 u^{1/3}\,du$

$= \frac{1}{2}\left(\frac{3}{4} u^{4/3} \Big|_1^5\right)$

$= \frac{3}{8}(5^{4/3} - 1)$ //

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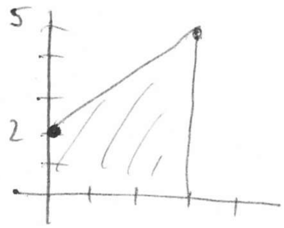
1. (6 pts.) Find the total displacement over the time interval $0 \le t \le 3$ of an object whose velocity function is given by $v(t) = t + 2$. Find also the total distance travelled over that time interval. (Hint: You don't have to use calculus to solve this problem.)



Disp $= 3\left(\frac{2+5}{2}\right) = \frac{21}{2}$ //

Distance = same $= \frac{21}{2}$ //

or Disp $= \int_0^3 t+2\,dt = \frac{1}{2}t^2+2t\Big|_0^3 = \frac{9}{2}+6 = \frac{21}{2}$

Distance $= \int_0^3 |t+2|\,dt = \int_0^3 t+2\,dt = \frac{21}{2}$.

2. (4 pts.) Evaluate the following definite integral using substitution. $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx$

$u = x^2+1$ $du = 2x\,dx$ $x=0$ $u=1$, $x=1$, $u=2$.

$\int_0^1 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_0^1 \frac{2x}{x^2+1}\,dx = \frac{1}{2}\int_1^2 \frac{du}{u}$

$= \frac{1}{2}\ln|u|\Big|_1^2 = \frac{1}{2}\ln(2) - \frac{1}{2}\ln(1) = \frac{1}{2}\ln 2$ //

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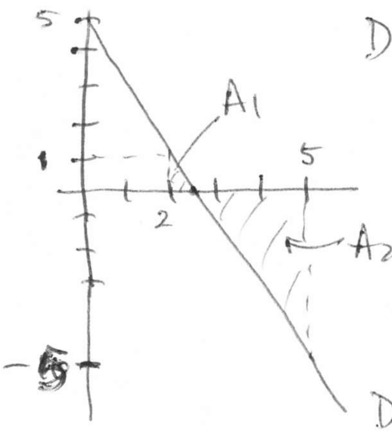
Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (6 pts.) Find the total displacement over the time interval $2 \le t \le 5$ of an object whose velocity function is given by $v(t) = 5 - 2t$. Find also the total distance travelled over that time interval. (Hint: You don't have to use calculus to solve this problem.)



$$\text{Displacement} = A_1 - A_2 = \tfrac{1}{2}\left(\tfrac{1}{2}\right)(1) - \tfrac{1}{2}\left(\tfrac{5}{2}\right)(5)$$

$$= \frac{1}{4} - \frac{25}{4} = -\frac{24}{4} = -6$$

$$\text{Total distance} = A_1 + A_2 = \frac{1}{4} + \frac{25}{4} = \frac{26}{4} = \frac{13}{2}$$

or $$\text{Disp} = \int_2^5 5 - 2t\,dt = 5t - t^2\Big|_2^5 = (25 - 25) - (10 - 4) = -6$$

$$\text{Dist} = \int_2^5 |5 - 2t|\,dt = \int_2^{5/2} 5 - 2t + \int_{5/2}^5 2t - 5$$

$$= (5t - t^2)\Big|_2^{5/2} + (t^2 - 5t)\Big|_{5/2}^5 = \frac{25}{2} - \frac{25}{4} - 10 + 4 + 25 - 25 - \frac{25}{4} + \frac{25}{2}$$

$$= 25 - \frac{25}{2} - 6 = \frac{13}{2}$$

2. (4 pts.) Evaluate the following definite integral using substitution. $\displaystyle\int_0^{\pi} x \sin(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$x = 0 \quad u = 0$

$x = \pi \quad u = \pi^2$

$$\int_0^{\pi} x \sin(x^2)\,dx = \frac{1}{2}\int_0^{\pi^2} \sin(u)\,du$$

$$= \frac{1}{2}(-\cos u)\Big|_0^{\pi^2} = \frac{1}{2}(-\cos \pi^2 + 1)$$

$$= \frac{1}{2}(1 - \cos(\pi^2))$$