

MATH 114 – 29 APRIL 2013 – EXAM 3

Answer each of the following questions on the sheets provided. Show all work, as partial credit may be given.

1. (5 pts. each) Consider the series  $\sum_{k=1}^{\infty} \left( \frac{1}{\sqrt{2k+1}} - \frac{1}{\sqrt{2k+3}} \right)$ .

- (a) Write out the first three terms of the *sequence of terms* for this series.
- (b) Write out the first three terms of the *sequence of partial sums* for this series.
- (c) Find a formula for  $s_n$ , the  $n^{\text{th}}$  partial sum of the series. (Hint: This is a telescoping series.)
- (d) Find the sum of the series.

2. (10 pts. each) Use the direct comparison or limit comparison test to determine if each of the following series converges. Justify your answer.

(a)  $\sum_{k=2}^{\infty} \sqrt{\frac{k^2}{k^4 + 1}}$

(b)  $\sum_{k=2}^{\infty} \frac{3^k}{10 + 5^k}$

3. (10 pts. each) Use the ratio or root test to determine if each of the following series converges.

(a)  $\sum_{k=2}^{\infty} \frac{k!}{(2k)!}$

(b)  $\sum_{k=2}^{\infty} \left( \frac{2k}{k+1} \right)^{k/2}$

4. (10 pts. each) Determine whether each of the following series converges or diverges by applying an appropriate convergence test. If the series converges, find its sum.

(b)  $\sum_{k=2}^{\infty} \frac{7}{4^k}$

(b)  $\sum_{k=1}^{\infty} \frac{k}{k + \sqrt{k}}$

5. (10 pts. each) Determine whether each of the following series is divergent, absolutely convergent or conditionally convergent. Justify your answer by applying an appropriate convergence test.

(a)  $\sum_{k=1}^{\infty} \frac{\sin(k^3)}{k^3}$

(b)  $\sum_{k=1}^{\infty} (-1)^k \frac{1}{k + \ln k}$