

MATH 114 – 29 APRIL 2013 – EXAM 3

Answer each of the following questions on the sheets provided. Show all work, as partial credit may be given.

1. (5 pts. each) Consider the series  $\sum_{k=1}^{\infty} \left( \frac{1}{k+1} - \frac{1}{k+2} \right)$ .

- (a) Write out the first three terms of the *sequence of terms* for this series.
- (b) Write out the first three terms of the *sequence of partial sums* for this series.
- (c) Find a formula for  $s_n$ , the  $n^{\text{th}}$  partial sum of the series. (Hint: This is a telescoping series.)
- (d) Find the sum of the series.

2. (10 pts. each) Use the direct comparison or limit comparison test to determine if the following series converge. Justify your answer.

(a)  $\sum_{k=2}^{\infty} \frac{k^2}{k^3 + 1}$

(b)  $\sum_{k=2}^{\infty} \frac{1}{1 + 3^k}$

3. (10 pts. each) Use the ratio or root test to determine if each of the following series converges.

(a)  $\sum_{k=2}^{\infty} \frac{10^{5k}}{k!}$

(b)  $\sum_{k=2}^{\infty} \left( \frac{k}{2k+1} \right)^{2k}$

4. (10 pts. each) Determine whether each of the following series converges or diverges by applying an appropriate convergence test. If the series converges, find its sum.

(a)  $\sum_{k=0}^{\infty} 8^{-k} 3^{k+1}$

(b)  $\sum_{k=1}^{\infty} \frac{k}{k + \ln(k)}$

5. (10 pts. each) Determine whether each of the following series is divergent, absolutely convergent or conditionally convergent. Justify your answer by applying an appropriate convergence test.

(a)  $\sum_{k=1}^{\infty} \frac{\cos(k^2)}{k^3}$

(b)  $\sum_{k=1}^{\infty} (-1)^k \frac{1}{k + \sqrt{k}}$