MATH 114 – EXAM 2 VERSION 1 – SOLUTIONS

1 (a) $\int_0^{\pi} x\sin(x)\,dx$ $\quad u = x \quad dv = \sin(x)\,dx$

$\quad du = dx \quad v = -\cos(x)$

$= -x\cos(x)\Big|_0^{\pi} + \int_0^{\pi}\cos(x)\,dx$

$= -\pi\cos\pi + 0\cdot\cos(0) + \sin(x)\Big|_0^{\pi} = \pi$ //

(b) $\int e^x\cos(x)\,dx$ $\quad u = e^x \quad dv = \cos(x)\,dx$

$\quad du = e^x\,dx \quad v = \sin(x)$

$= e^x\sin(x) - \int e^x\sin(x)\,dx$ $\quad u = e^x \quad dv = \sin(x)dx$

$\quad du = e^x dx \quad v = -\cos(x)$

$= e^x\sin(x) - \left[-e^x\cos(x) + \int e^x\cos(x)\,dx\right]$

$= e^x\sin(x) + e^x\cos(x) - \int e^x\cos(x)\,dx$

$\therefore 2\int e^x\cos(x)\,dx = e^x(\sin(x) + \cos(x))$

$\therefore \int e^x\cos(x)\,dx = \frac{1}{2}e^x(\sin(x) + \cos(x)) + C$

U1

2. $\displaystyle\int \frac{dx}{(16+x^2)^{3/2}}$

$x = 4\tan\theta$

$dx = 4\sec^2\theta\, d\theta$

$16+x^2 = 16+16\tan^2\theta$

$= 16(1+\tan^2\theta)$

$= 16\sec^2\theta$

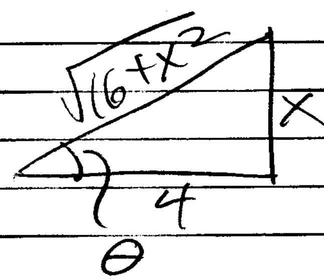
$\displaystyle = \int \frac{4\sec^2\theta}{(16\sec^2\theta)^{3/2}}\, d\theta$

$\displaystyle = \int \frac{4\sec^2\theta}{64\sec^3\theta}\, d\theta = \frac{1}{16}\int \frac{1}{\sec\theta}\, d\theta$

$\displaystyle = \frac{1}{16}\int \cos\theta\, d\theta = \frac{1}{16}\sin(\theta) + C.$

$\theta = \tan^{-1}\left(\frac{x}{4}\right)$

$\displaystyle = \frac{1}{16}\frac{x}{\sqrt{16+x^2}} + C$ //



3. $\displaystyle\int_1^2 \frac{2x-3}{x(x+1)^2}\,dx$ | $\displaystyle\frac{2x-3}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$$2x-3 = A(x+1)^2 + Bx(x+1) + Cx$$

$\underline{x=0}$ $\quad -3 = A \longrightarrow A = -3$ //

$\underline{x=-1}$ $\quad -5 = -C \longrightarrow C = 5$ //

$\underline{x=1}$ $\quad -1 = 4A + 2B + C$

$$= -12 + 2B + 5$$

$$= -7 + 2B \longrightarrow B = 3$$ //

$$\int_1^2 \frac{2x-3}{x(x+1)^2}\,dx = -3\int_1^2 \frac{1}{x}\,dx + 3\int_1^2 \frac{1}{x+1}\,dx + 5\int_1^2 \frac{dx}{(x+1)^2}$$

$$= -3\ln|x|\Big|_1^2 + 3\ln|x+1|\Big|_1^2 - \frac{5}{(x+1)}\Big|_1^2$$

$$= -3(\ln 2 - \overset{0}{\cancel{\ln 1}}) + 3(\ln(3) - \ln(2)) - \left(\frac{5}{3} - \frac{5}{2}\right)$$

$$= 3\ln 3 - 6\ln 2 + \frac{5}{6}$$ //

4. (a) $\int \sin^3(x)\cos^2(x)\,dx$

$= \int \sin^2(x)\cos^2(x)\sin(x)\,dx$

$= \int (1-\cos^2(x))\cos^2(x)\sin(x)\,dx$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$= -\int (1-u^2)u^2\,du$

$= \int u^4 - u^2\,du = \frac{1}{5}u^5 - \frac{1}{3}u^3 + c$

$= \frac{1}{5}\cos^5(x) - \frac{1}{3}\cos^3(x) + C$ //

(b) $\int \tan^4(x)\,dx = \int \tan^2(x)(\sec^2(x)-1)\,dx$

$= \int \tan^2(x)\sec^2(x)\,dx - \int \tan^2(x)\,dx$

$= \int \tan^2(x)\sec^2(x)\,dx - \int \sec^2(x)\,dx + \int dx$

$u = \tan x$

$du = \sec^2 x$

$\rightarrow \int u^2\,du = \frac{1}{3}u^3 + c$

$= \frac{1}{3}\tan^3 x - \tan x + x + C$ //

5. (a) $\int_0^\infty e^{-2x}\,dx = \lim_{b\to\infty} \int_0^b e^{-2x}\,dx$

$= \lim_{b\to\infty} -\frac{1}{2}e^{-2x}\Big|_0^b = \lim_{b\to\infty}\left(-\frac{1}{2}e^{-2b} + \frac{1}{2}\right) = \frac{1}{2}$

(b) $\int_0^1 x^{-5/2}\,dx = \lim_{c\to 0^+} \int_c^1 x^{-5/2}\,dx$

$= \lim_{c\to 0^+} -\frac{2}{3}x^{-3/2}\Big|_c^1 = \lim_{c\to 0^+}\left(-\frac{2}{3} + \frac{2}{3}c^{-3/2}\right)$

$= \infty$ (integral diverges)

MATH 114 – EXAM 2 VERSION 2 – SOLUTIONS

1. (a) $\int_1^2 \frac{\ln x}{x^3}\,dx$ $\quad u = \ln x \quad dv = x^{-3}dx$

$du = \frac{1}{x}dx \quad v = -\frac{1}{2}x^{-2}$

$= -\frac{1}{2}x^{-2}\ln x \Big|_1^2 + \frac{1}{2}\int_1^2 x^{-3}\,dx$

$= -\frac{1}{8}\ln(2) + \cancel{\frac{1}{2}\ln(1)}^{\,0} + \frac{1}{2}\left[-\frac{1}{2}x^{-2}\Big|_1^2\right]$

$= -\frac{1}{8}\ln(2) - \frac{1}{16} + \frac{1}{4} = -\frac{1}{8}\ln(2) + \frac{3}{16}$ //

(b) $\int e^x \sin(x)\,dx$ $\quad u = \sin(x) \quad dv = e^x dx$

$du = \cos(x)\,dx \quad v = e^x$

$= e^x\sin(x) - \int e^x\cos(x)\,dx$ $\quad u = \cos(x) \quad v = e^x dx$

$du = -\sin(x)dx \quad v = e^x$

$= e^x\sin(x) - \left[e^x\cos(x) + \int e^x \sin(x)\,dx\right]$

$= e^x(\sin(x) - \cos(x)) - \int e^x\sin(x)\,dx$

$2\int e^x\sin(x)\,dx = e^x(\sin(x) - \cos(x)) + C$

$\int e^x\sin(x)\,dx = \frac{1}{2}e^x(\sin(x) - \cos(x)) + C$ //

V2

2. $\displaystyle\int \frac{dx}{x^2(x^2-9)^{1/2}}$

$x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

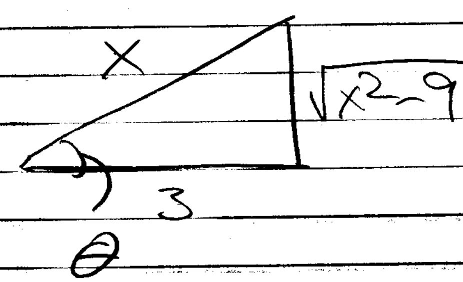
$x^2 - 9 = 9\sec^2\theta - 9$

$= 9(\sec^2\theta - 1) = 9\tan^2\theta$

$$= \int \frac{3\sec\theta\tan\theta}{9\sec^2\theta \cdot 3\tan\theta}\,d\theta = \frac{1}{9}\int \frac{1}{\sec\theta}\,d\theta$$

$$= \frac{1}{9}\int \cos\theta\,d\theta = \frac{1}{9}\sin\theta + C \qquad \theta = \sec^{-1}\left(\frac{x}{3}\right)$$

$$= \frac{1}{9}\,\frac{\sqrt{x^2-9}}{x} + C \quad //.$$



3. $\displaystyle\int_2^3 \frac{x}{(x-1)(x+2)^2}\,dx$ | $\displaystyle\frac{x}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

$$x = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$\underline{x=1}$ $\quad 1 = 9A \longrightarrow A = \frac{1}{9}$ //

$\underline{x=-2}$ $\quad -2 = -3C \longrightarrow C = \frac{2}{3}$ //

$\underline{x=0}$ $\quad 0 = 4A - 2B - C = \frac{4}{9} - 2B - \frac{2}{3}$

$$= -\frac{2}{9} - 2B \longrightarrow B = \frac{-1}{9} //.$$

$$= \frac{1}{9}\int_2^3 \frac{1}{x-1}\,dx - \frac{1}{9}\int_2^3 \frac{1}{x+2}\,dx + \frac{2}{3}\int_2^3 \frac{1}{(x+2)^2}\,dx$$

$$= \frac{1}{9}\ln|x-1|\Big|_2^3 - \frac{1}{9}\ln|x+2|\Big|_2^3 - \frac{2}{3}\frac{1}{(x+2)}\Big|_2^3$$

$$= \frac{1}{9}(\ln(2) - \cancelto{0}{\ln(1)}) - \frac{1}{9}(\ln 5 - \ln 4) - \frac{2}{3}\left[\frac{1}{5} - \frac{1}{4}\right]$$

$$= \frac{1}{9}\ln(2) - \frac{1}{9}\ln\left(\frac{5}{4}\right) + \frac{2}{3}\cdot\frac{1}{20}$$

$$= \frac{1}{9}\ln\left(\frac{8}{5}\right) + \frac{1}{30} //$$

4. (a) $\int \cos^3(x)\sin^4(x)\,dx$

$= \int \cos^2(x)\sin^4(x)\cos(x)\,dx$

$= \int (1-\sin^2(x))\sin^4(x)\cos(x)\,dx$ $\quad u=\sin(x)$, $du=\cos(x)\,dx$

$= \int (1-u^2)u^4\,du = \int u^4-u^6\,du$

$= \frac{1}{5}u^5 - \frac{1}{7}u^7 + C$

$= \frac{1}{5}\sin^5(x) - \frac{1}{7}\sin^7(x) + C$ //

(b) $\int \sec^4(x)\,dx = \int \sec^2(x)(\tan^2(x)+1)\,dx$

$= \int \tan^2(x)\sec^2(x)\,dx + \int \sec^2(x)\,dx$

$\downarrow$ $\quad u=\tan x$, $du=\sec^2 x\,dx$

$\int u^2\,du = \frac{1}{3}u^3 + C$

$= \frac{1}{3}\tan^3(x) + \tan(x) + C$ //

5. (a) $\displaystyle\int_2^{\infty} \frac{\ln(x)}{x^3}\,dx = \lim_{b\to\infty} \int_2^{b} \frac{\ln(x)}{x^3}\,dx$

$$= \lim_{b\to\infty} \Big(\underbrace{-\tfrac{1}{2}x^{-2}\ln x - \tfrac{1}{4}x^{-2}}_{\text{from Problem 1(a)}} \Big|_2^b \Big)$$

$$= \lim_{b\to\infty} \Big(-\frac{1}{2}\cancel{\frac{\ln(b)}{b^2}}^{\,0} - \cancel{\frac{1}{4}b^{-2}}^{\,0} - \Big(\frac{1}{8}\ln(2) - \frac{1}{16}\Big)\Big)$$

$$= \frac{1}{16} - \frac{1}{8}\ln(2) \; //.$$

(b) $\displaystyle\int_0^2 x^{-2/5}\,dx = \lim_{c\to 0^+} \int_c^2 x^{-2/5}\,dx$

$$= \lim_{c\to 0^+} \frac{5}{3}x^{3/5}\Big|_c^2 = \lim_{c\to 0^+} \Big(\frac{5}{3}(2)^{3/5} - \frac{5}{3}c^{3/5}\Big)$$

$$= \frac{5}{3}2^{3/5} \; //.$$