MATH 114 – EXAM 1 – VERSION 1 – SOLUTIONS

1. (a) $s(5)-s(0)=\int_0^5 t^3-2t^2\,dt=\frac{1}{4}t^4-\frac{2}{3}t^3\Big|_0^5$

$=\frac{1}{4}(5)^4-\frac{2}{3}(5)^3=\frac{625}{4}-\frac{250}{3}=\frac{875}{12}$ m.

≈ 73 m. //

(b) $t^3-2t^2=0$

$t^2(t-2)=0$

velocity $=0$ at $t=0, t=2$

$\int_0^5 |t^3-2t^2|\,dt=\int_0^2 2t^2-t^3\,dt+\int_2^5 t^3-2t^2\,dt$

$=\left(\frac{2}{3}t^3-\frac{1}{4}t^4\Big|_0^2\right)+\left(\frac{1}{4}t^4-\frac{2}{3}t^3\Big|_2^5\right)$

$=\frac{16}{3}-4+\frac{625}{4}-\frac{250}{3}-4+\frac{16}{3}$

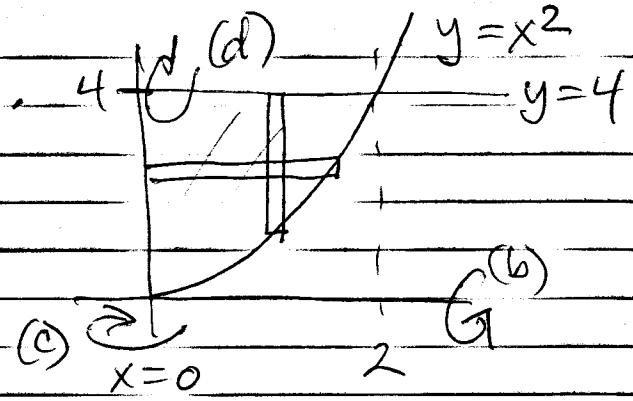
$=\frac{-218}{3}+\frac{625}{4}-8=\frac{907}{12}\approx 75.5$ m. //

(c) $s(t)=\int_0^t v(x)\,dx+s(0)$

$=\left(\int_0^t x^3-2x^2\,dx\right)-2$

$=\frac{1}{4}t^4-\frac{2}{3}t^3-2$ //

2.



(a) $A=\int_0^2 4-x^2\,dx$

$=4x-\frac{1}{3}x^3\Big|_0^2$

$=8-\frac{8}{3}=\frac{16}{3}$ //

~~(b)~~ OR $A=\int_0^4 y^{1/2}\,dy=\frac{2}{3}y^{3/2}\Big|_0^4=\frac{2}{3}(4)^{3/2}=\frac{16}{3}$ //

(b) $dV=\pi(4^2-(x^2)^2)\,dx$

$V=\int_0^2 \pi(16-x^4)\,dx=\pi\left(16x-\frac{1}{5}x^5\Big|_0^2\right)$

$=\pi\left(32-\frac{32}{5}\right)=\frac{128\pi}{5}$ //

(c) $dV=2\pi x(4-x^2)\,dx$

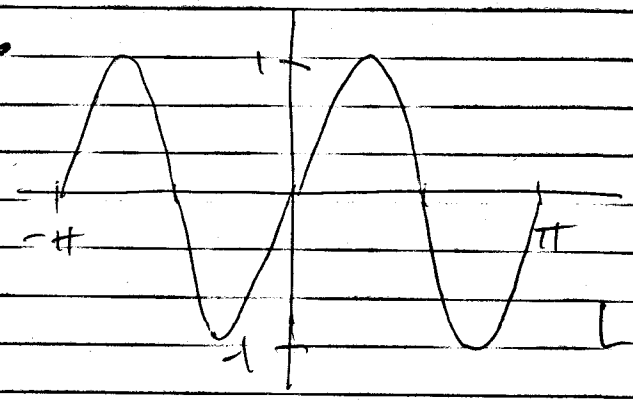
$V=\int_0^2 2\pi(4x-x^3)\,dx=2\pi\left(2x^2-\frac{1}{4}x^4\Big|_0^2\right)$

$=2\pi(8-4)=8\pi$ //

(d) washers: $V=\int_0^2 \pi(4-x^2)^2\,dx=\pi\int_0^2 16-8x^2+x^4\,dx$

shells: $V=\int_0^4 2\pi(4-y)(y^{1/2})\,dy=2\pi\int_0^4 4y^{1/2}-y^{3/2}\,dy$

3.



$y = \sin(2x)$

$y' = 2\cos(2x)$

$$L = \int_{-\pi}^{\pi} \sqrt{1+(y')^2}\,dx$$

$$= \int_{-\pi}^{\pi} \left(1+4\cos^2(2x)\right)^{1/2} dx$$ //

4. $\int_0^1 \frac{2x-1}{x+2}\,dx$ $\quad u = x+2$

$du = dx$

$x=0 \quad u=2$

$x=1 \quad u=3$

$2x-1 = 2(u-2)-1 = 2u-5$

$$= \int_2^3 \frac{2u-5}{u}\,du$$

$$= \int_2^3 2 - \frac{5}{u}\,du = 2u - 5\ln|u| \Big|_2^3$$

$$= 6 - 5\ln(3) - 4 + 5\ln(2)$$

$$= 2 + 5\ln(2/3)$$ //

5. $P(t) = P_0 e^{rt} = 100 e^{.03t}$

$P(10) = 100 e^{.03(10)} = 100 e^{.3} \approx$

OR

$P(10) = 100 (1.03)^2 \approx$

MATH 114 – EXAM 1 – VERSION 2 – SOLUTIONS

1. (a) $S(5) - S(0) = \int_0^5 4t - t^2 \, dt$

$= 2t^2 - \frac{1}{3}t^3 \Big|_0^5 = 50 - \frac{125}{3} = \frac{25}{3} \approx 8.3$ m //

(b) $4t - t^2 = 0$

$t(4-t) = 0$

$t = 0 \quad t = 4$ ← velocity $= 0$.

$\text{Dist} = \int_0^5 |4t - t^2| \, dt = \int_0^4 4t - t^2 \, dt + \int_4^5 t^2 - 4t \, dt$

$= 2t^2 - \frac{1}{3}t^3 \Big|_0^4 + \frac{1}{3}t^3 - 2t^2 \Big|_4^5$

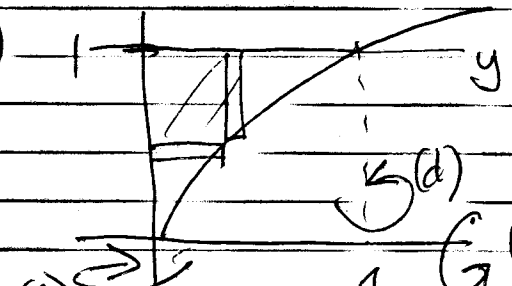
$= 32 - \frac{64}{3} + \frac{125}{3} - 50 - \frac{64}{3} + 32$

$= 14 - 1 = 13$ m. //

(c) $S(t) = \int_0^t 4x - x^2 \, dx + S(0)$

$= 4t^2 - \frac{1}{3}t^3 - 4$ //

2.



(a) $A = \int_0^1 1 - x^{1/3}\,dx$

$= x - \frac{3}{4}x^{4/3}\Big|_0^1$

$= 1 - \frac{3}{4} = \frac{1}{4}$ //

OR $\int_0^1 y^3\,dy = \frac{1}{4}y^4\Big|_0^1 = \frac{1}{4}$ //

(b) $dV = \pi\left(1^2 - (x^{1/3})^2\right)dx$

$V = \int_0^1 \pi\left(1 - x^{2/3}\right)dx = \pi\left(x - \frac{3}{5}x^{5/3}\Big|_0^1\right)$

$= \pi\left(1 - \frac{3}{5}\right) = \frac{2\pi}{5}$ //

(c) $dV = 2\pi(x)\left(1 - x^{1/3}\right)dx$

$V = 2\pi\int_0^1 x - x^{4/3}\,dx = 2\pi\left(\frac{1}{2}x^2 - \frac{3}{7}x^{7/3}\Big|_0^1\right)$

$= 2\pi\left(\frac{1}{2} - \frac{3}{7}\right) = \frac{\pi}{7}$ //

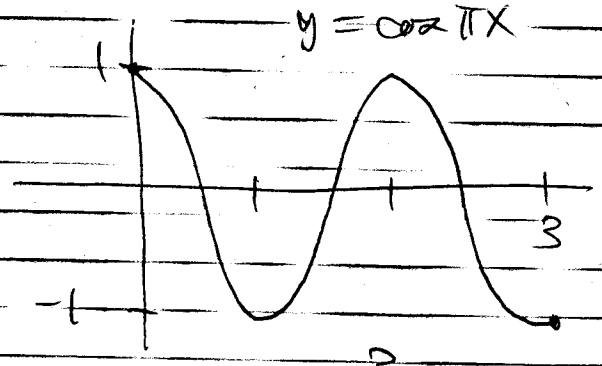
(d) Washers: $V = \int_0^1 \pi\left(1^2 - (1-y^3)^2\right)dy$

$= \int_0^1 \pi\left(1 - (1 - 2y^3 + y^6)\right)dy$

$= \int_0^1 \pi\left(2y^3 - y^6\right)dy$

Shells: $V = \int_0^1 2\pi(1-x)\left(1 - x^{1/3}\right)dx$

$= \int_0^1 2\pi\left(1 - x^{1/3} - x + x^{4/3}\right)dx$

3. 

$y' = -\pi \sin \pi x$

$1 + (y')^2 = 1 + \pi^2 \sin^2(\pi x)$

$$L = \int_0^3 \left(1 + \pi^2 \sin^2(\pi x)\right)^{1/2} dx \quad //$$

4. $\frac{1}{2}\int_1^2 \frac{x+2}{2x-1}(2\,dx)$ $\quad u = 2x-1$

$du = 2\,dx$

$x+2 = \left(\frac{u+1}{2}\right)+2 = \frac{1}{2}(u+5)$

$x = 1 \quad u = 1$

$x = 2 \quad u = 3$

$$= \frac{1}{2}\int_1^3 \frac{1}{2}\frac{u+5}{u}\,du = \frac{1}{4}\int_1^3 1 + \frac{5}{u}\,du$$

$$= \frac{1}{4}\left(u + 5\ln|u|\,\Big|_1^3\right) = \frac{1}{4}\left(3 + 5\ln(3) - 1 - 5\cancel{\ln(1)}^{0}\right)$$

$$= \frac{1}{4}(2 + 5\ln 3) \quad //$$

5. $P(t) = (1.5)\,2^{t/12} = (1.5)\,e^{\frac{\ln 2}{12}t}$

$\approx (1.5)\,e^{.057t}$ //