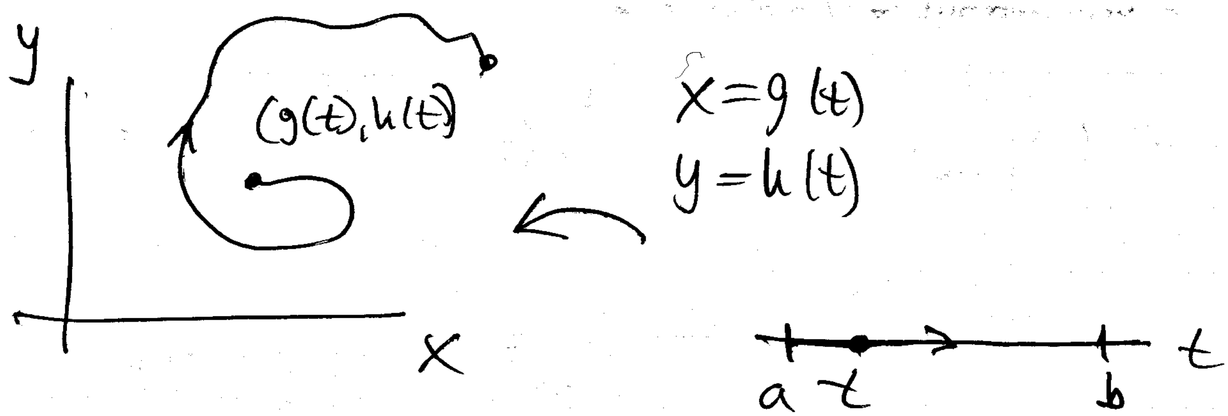
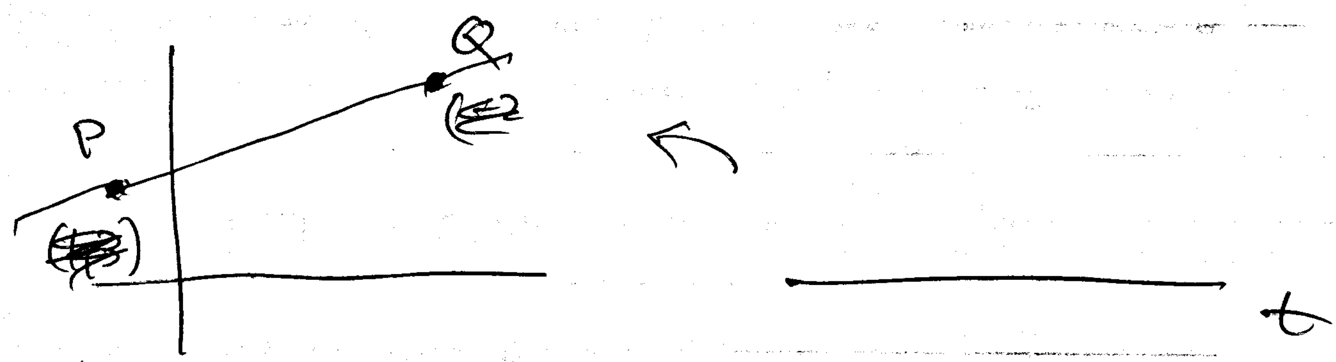
Parametric Equations



e.g. $x = 4 - 3t \quad y = -2 + 6t$

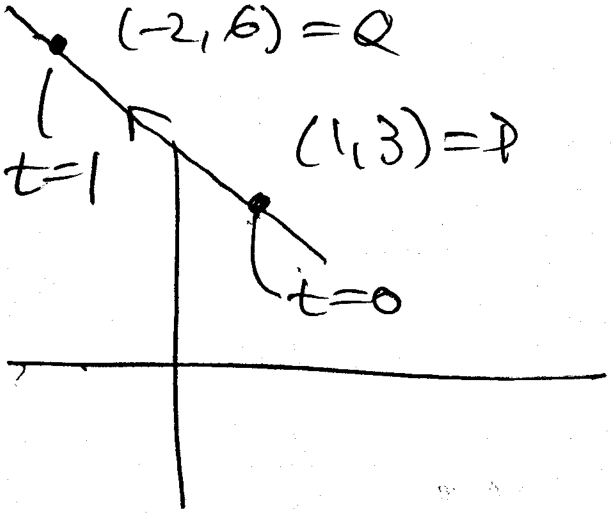
curve: line with slope $\frac{6}{-3} = -2$.

Given 2 points, find parametric equations for line joining the points:



e.g. $P = (1, 3) \quad Q = (-2, 6)$

$x =$ $\qquad$ $y =$



Look at:

$$\boxed{(1,3)(1-t)+(-2,6)t}$$

$$(1\cdot(1-t),3(1-t))+$$
$$(-2t,6t)$$

$$(1-t-2t,3(1-t)+6t)$$

$$\boxed{\begin{aligned}x&=1-3t\\ y&=3+3t\end{aligned}}$$

$$(x,y)=(1-3t,3+3t)$$

$t=0$: $x=1$ $y=3$

$t=1$: $x=-2$ $y=6$

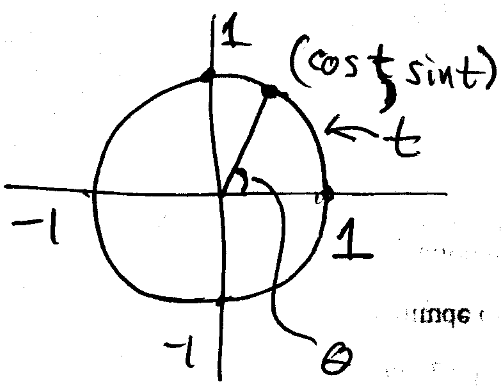
<u>e.g</u> $P=(-8,2)$ $Q=(1,2)$

$$(-8,2)(1-t)+(1,2)(t)$$

$$(-8)(1-t)+(1)(t)=x\to \boxed{x=-8+9t}$$

$$2(1-t)+2t=y\to \boxed{y=2}$$

circles



parametric rep of a circle (radius 1).

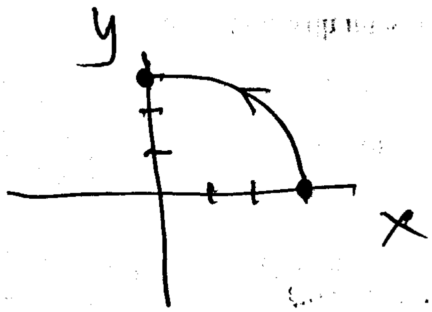
$$x = \cos t, \quad y = \sin t$$

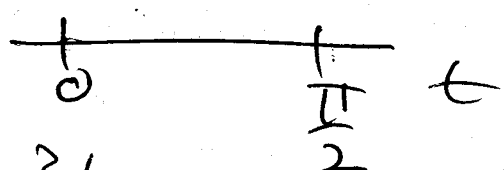
$$0 \le t \le 2\pi$$

parametrizes unit circle

$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

(6) $x = 3\cos t \;,\; y = 3 \cdot \sin t \quad 0 \le t \le \frac{\pi}{2}$





$$x^2 + y^2 = 9\cos^2 t + 9\sin^2 t$$

$$= 9(\cos^2 t + \sin^2 t) = 9$$

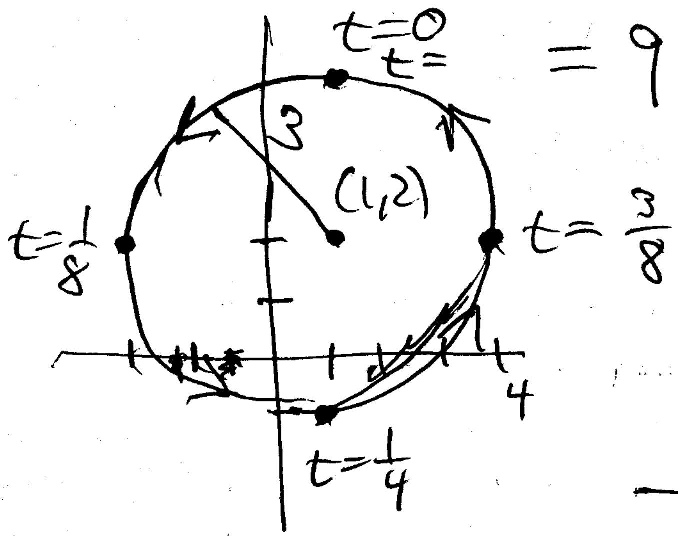
18) $x = 1 - 3\sin(4\pi t)$ $y = 2 + 3\cos(4\pi t)$

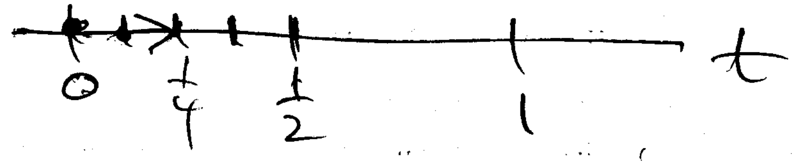
$0 \le t \le \frac{1}{2}$

$x - 1 = -3\sin(4\pi t)$ $y - 2 = 3\cos(4\pi t)$

$(x-1)^2 + (y-2)^2 = 9\sin^2(4\pi t) + 9\cos^2(4\pi t)$

$= 9$





$t=0$: $x = 1$ $y = 5$

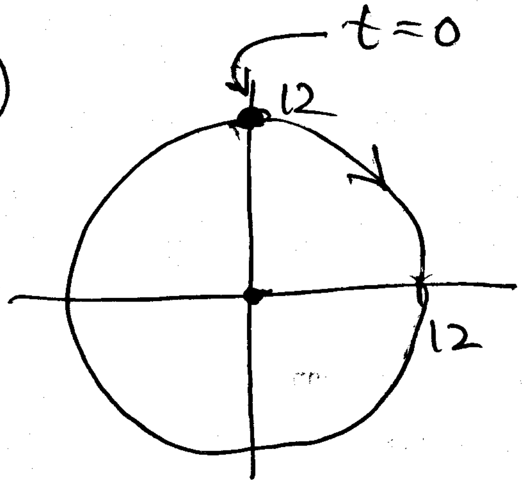
$t=\frac{1}{4}$: $x = 1$ $y = -1$

$1 - 3\sin(4\pi \cdot \frac{1}{4}) = 1 - 3\sin\pi = 1$

$2 + 3\cos(4\pi \cdot \frac{1}{4}) = 2 + 3\cos\pi = -1$

$t = \frac{1}{8}$: $x = -2$ $y = 2$

#20)



Trying

$x = 12 \cancel{\cos t}$

$y = 12 \cancel{\sin t}$

initial pt: $t=0 \to (12,0)$

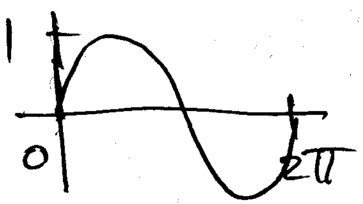
NO GOOD

$x = 12 \sin t$

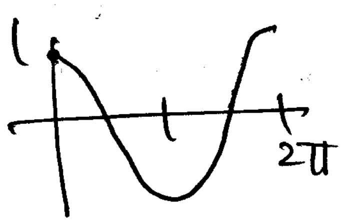
$y = 12 \cos t$

correct initial point.

orientation: <u>OK</u>.



sin t



cos t

$x = 12 \sin t$

$y = -12 \cos t$

opposite orientation but wrong starting point.

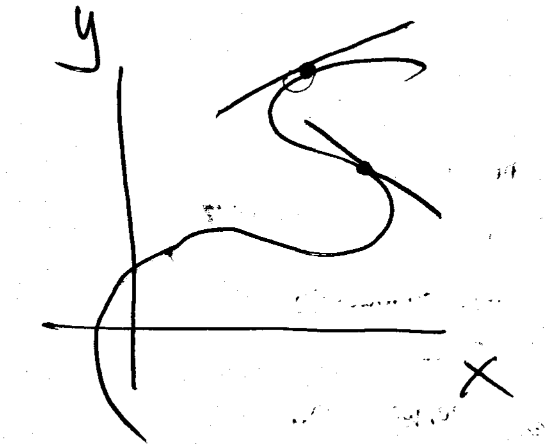
NO GOOD

$x = -12 \sin t$

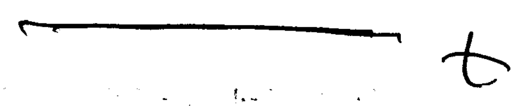
$y = 12 \cos t$

✓

Derivatives



$x=g(t)$
$y=h(t)$

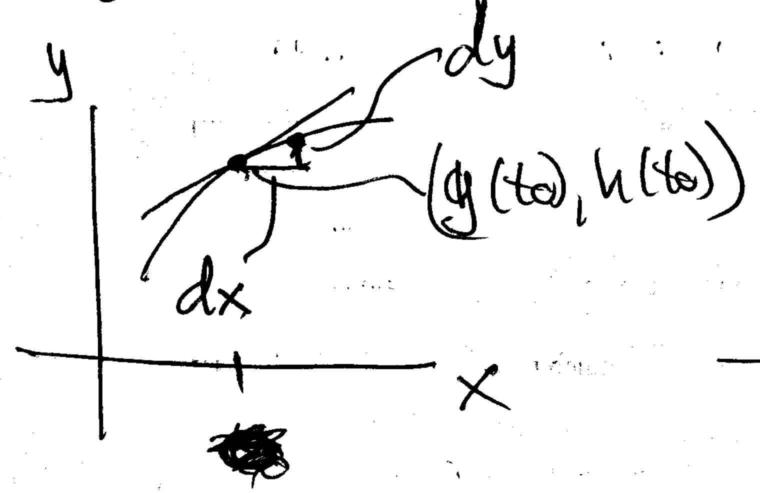


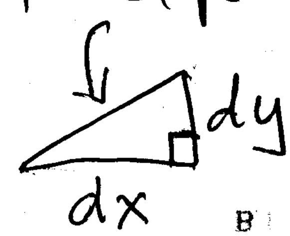
Slope of tangent line to curve?

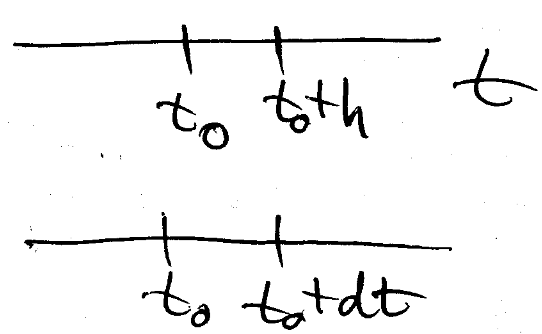
e.g. $x^3-6xy+y^3=0$ Find $\frac{dy}{dx}$

Implicit differentiation

What about parametric? slope $=\frac{dy}{dx}$







$x=g(t)$

$dx=g'(t)\,dt$

$dy=h'(t)\,dt$

Slope is $\frac{dy}{dx} = \frac{h'(t)\,dt}{g'(t)\,dt} = \frac{h'(t)}{g'(t)}$

OR $\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{h'(t)}{g'(t)}}$

48) $x = 2t \quad y = t^3 \quad t = -1$

Find slope of tangent line:

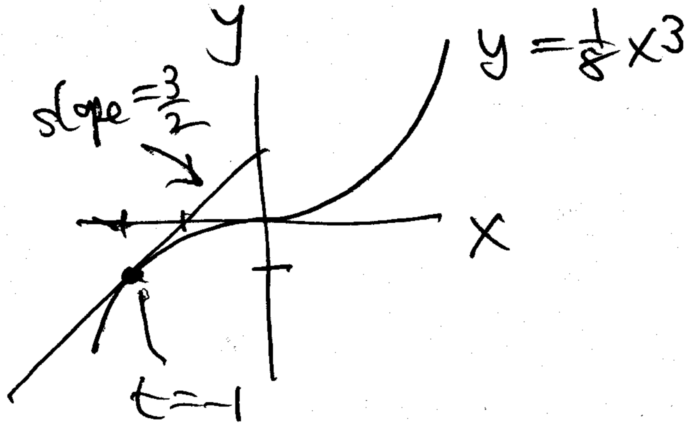
$\frac{dy}{dt} = 3t^2 \quad \frac{dx}{dt} = 2$

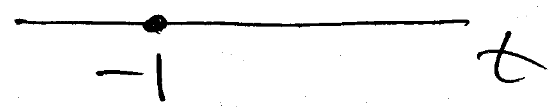
$\frac{dy}{dx} = \frac{3t^2}{2} = \frac{3}{2}t^2$

$\left.\frac{dy}{dx}\right|_{t=-1} = \frac{3}{2}(-1)^2 = \frac{3}{2}$

$t = \frac{x}{2} \rightarrow y = t^3 \rightarrow y = \left(\frac{x}{2}\right)^3$

$y = \frac{1}{8}x^3$

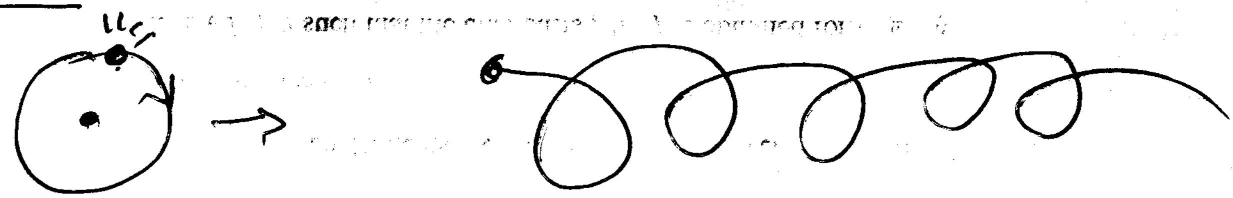


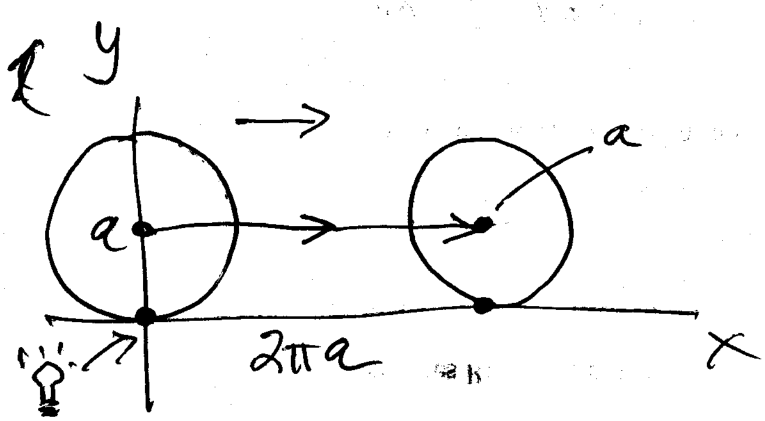


parameter space

$t = -1$: $x = -2,\ y = -1$

Cycloid





to go $2\pi a$ ~~distance~~ feet takes 2π seconds

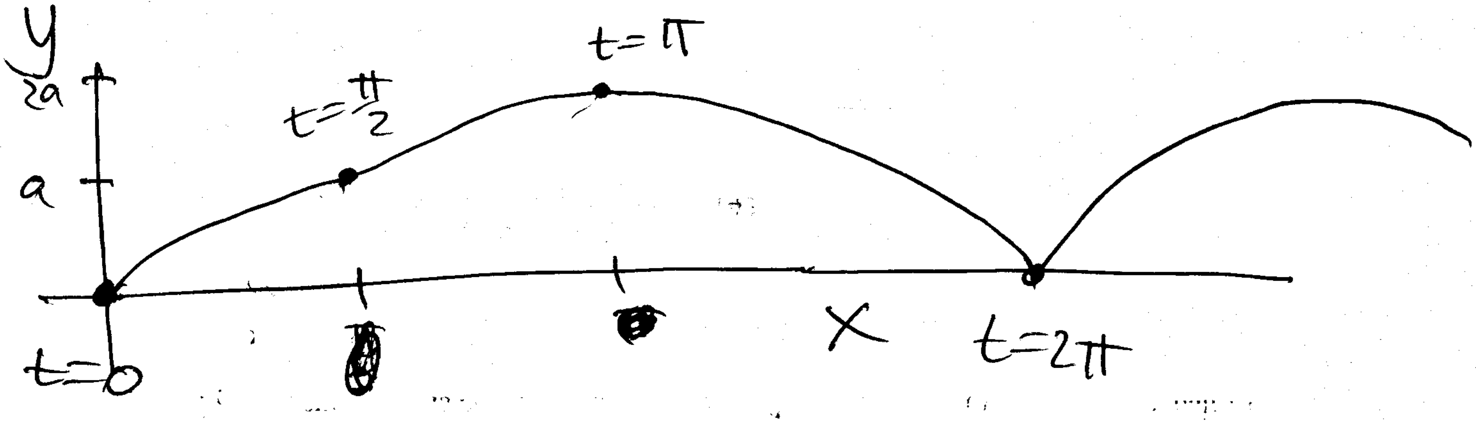
so rate $= a\ \frac{\text{feet}}{\text{sec}}$.

radius $= a$ $\qquad x = at - a\sin t$

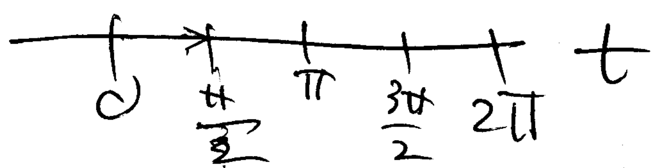
center $= (at, a)$ $\qquad y = a - a\cos t$

$$x = a(t - \sin t)$$
$$y = a(1 - \cos t)$$

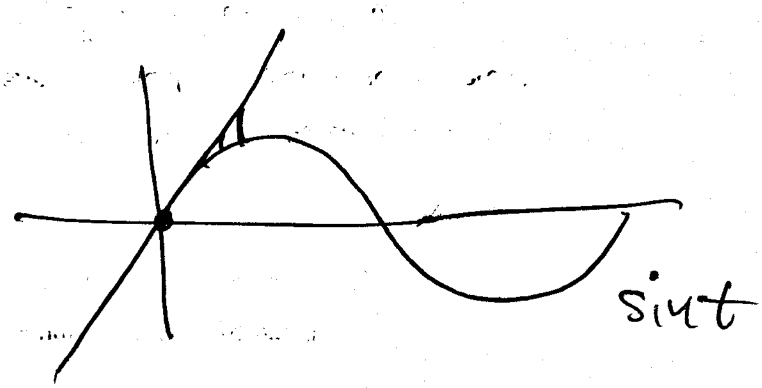
$$0 \le t \le 2\pi$$



x does not go backwards.

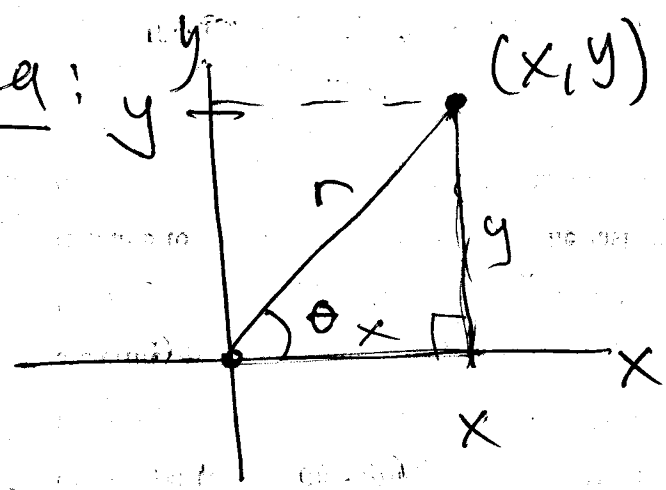


$t - \sin t > 0$ if $t > 0$.



10.2 Polar coordinates

Idea:



Alternate representation of (x,y) as (r,θ)

$r \geq 0,\ 0 \leq \theta < 2\pi$

$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = x^2 + y^2$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Plotting polar graphs