Quiz 14 – 9.3, [redacted]

Final Exam – Wed 5/8

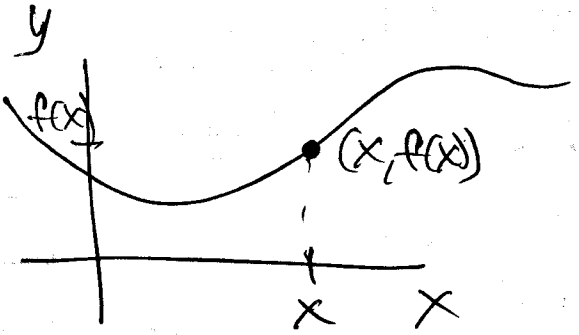
About ½ on Ch 9, 10

About ½ on rest of semester.

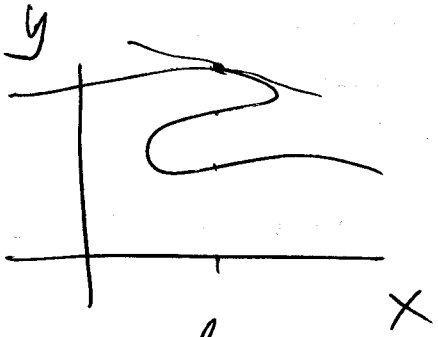
10.1 Parametric Equations

Q: How do you represent a curve in the plane?

① As graph of $y = f(x)$

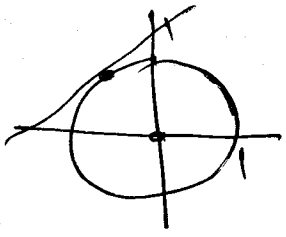


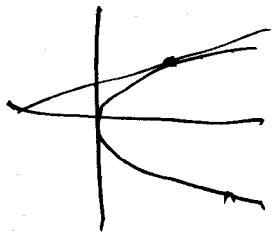
Does not always work.



② As an equation in x and y.

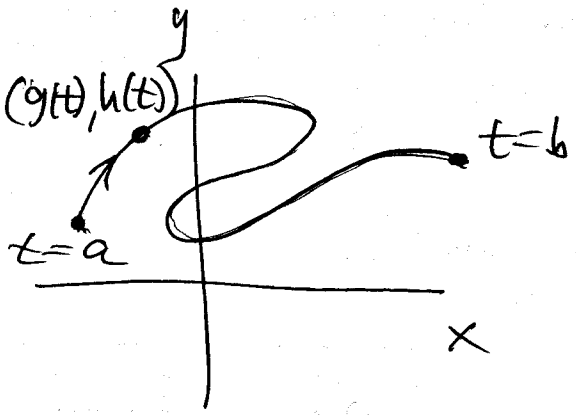
$x^2 + y^2 = 1$ $\quad x = y^2$ $\quad x^3 + 2xy + y^3 = 0$



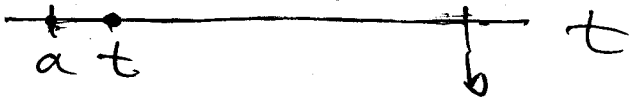


③ Parametrically

Idea: Think of curve as a map or an image of a line or line segment.



$x = g(t)$
$y = h(t)$



x-y plane

parameter space

Why useful? ① You can give curves an orientation ② Useful for describing motion (usu. t = time, $(x, y) = (g(t), h(t))$ = position)

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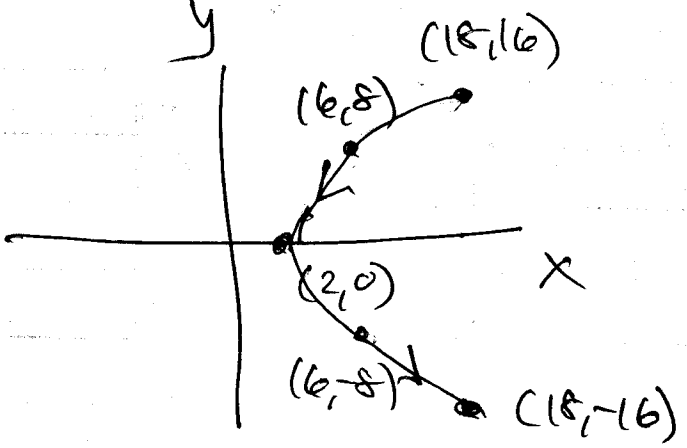
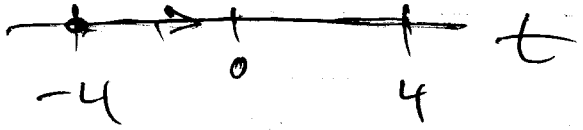
e.g. #8) $x = t^2 + 2$ $y = -4t$ $-4 \le t \le 4$





$t = -4$: $x = 18$ $y = 16$

$t = 0$: $x = 2$ $y = 0$

$t = 4$: $x = 18$ $y = -16$

$t = -2$: $x = 6$ $y = 8$

$t = 2$: $x = 6$ $y = -8$

Sometimes you can find an equation for the curve: Eliminating the parameter.

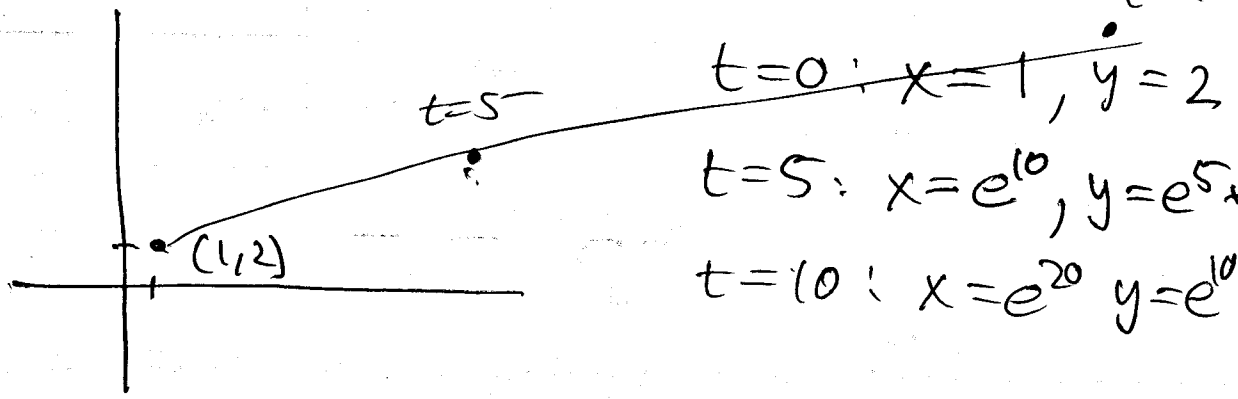
$x = t^2 + 2 \qquad y = -4t$

$\rightarrow$ solve for t: $t = -\frac{y}{4}$

Plug into here

$x = \left(-\frac{y}{4}\right)^2 + 2 \longrightarrow x = \frac{y^2}{16} + 2$

#14) $x = e^{2t} \qquad y = e^t + 1 \qquad 0 \le t \le 25$



$t=0$: $x=1$, $y=2$

$t=5$: $x=e^{10}$, $y=e^5+1$

$t=10$: $x=e^{20}$ $y=e^{10}+1$

$x = e^{2t} = (e^t)^2 \qquad y = e^t + 1$

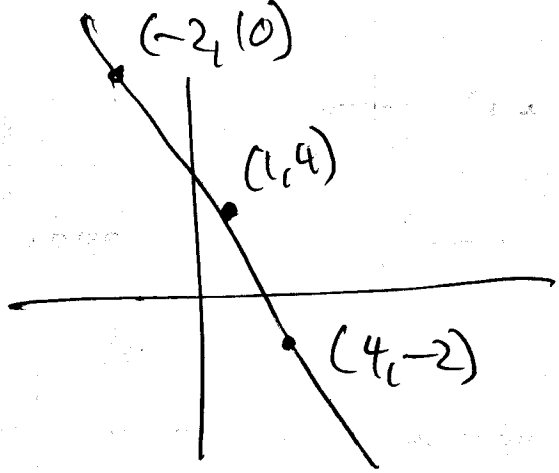
$\rightarrow e^t = y - 1$

$x = (y-1)^2$

Line segments.

#28) $x = 4 - 3t \quad y = -2 + 6t$

Each equation is a line, so curve is a line.



$t = 0 \rightarrow (4, -2)$

$t = 1 \rightarrow (1, 4)$

$t = 2 \rightarrow (-2, 10)$

$\text{slope} = \frac{6}{-3} = -2$