Quiz 7 – Thursday 7.4 (Partial Fractions)

Quiz 6: $\int \tan^2(x)\sec(x)\,dx$

$$= \int \frac{\sin^2(x)}{\cos^2(x)} \cdot \frac{1}{\cos(x)}\,dx = \int \frac{\sin^2(x)}{\cos^3(x)}\,dx$$

$$= \int \frac{\sin^2(x)}{\cos^4(x)} \cos(x)\,dx = \int \frac{\sin^2(x)}{(1-\sin^2(x))^2} \cos(x)\,dx$$

$u = \sin(x)$

$du = \cos(x)\,dx$

$$= \int \frac{u^2}{(1-u^2)^2}\,du$$

$$(1-u^2)^2 \to ((1-u)(1+u))^2$$

$$\frac{u^2}{(1-u)^2(1+u)^2} = \frac{A}{1-u} + \frac{B}{(1-u)^2} + \frac{C}{1+u} + \frac{D}{(1+u)^2}$$

This can be solved but is a lot of work!

$$\int \tan^3(x)\sec(x)\,dx = \int \tan^2(x)\,\tan(x)\sec(x)\,dx$$

$$= \int (\sec^2(x) - 1)\,\sec(x)\tan(x)\,dx$$

$u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$$= \int (u^2 - 1)\,du$$

$$= \frac{1}{3}u^3 - u + C = \frac{1}{3}\sec^3(x) - \sec(x) + C$$

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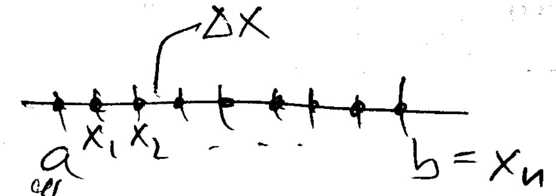
Numerical Integration

Trapezoid Rule
Midpoint Rule
Simpson's Rule

$$\left.\begin{matrix}\text{Trapezoid Rule}\\ \text{Midpoint Rule}\\ \text{Simpson's Rule}\end{matrix}\right\} \int_a^b f(x)\,dx$$



x_0

$$\Delta x = \left(\frac{b-a}{n}\right)$$

Find $\underline{\underline{y_i}} = f(x_i)$

Errors:

① Percentage error is more informative than absolute error.

② E_T and E_M ~~are~~ are approximately ~~dependent~~ proportional to on $(\Delta x)^2$

③ E_S is approx. proportional to $(\Delta x)^4$

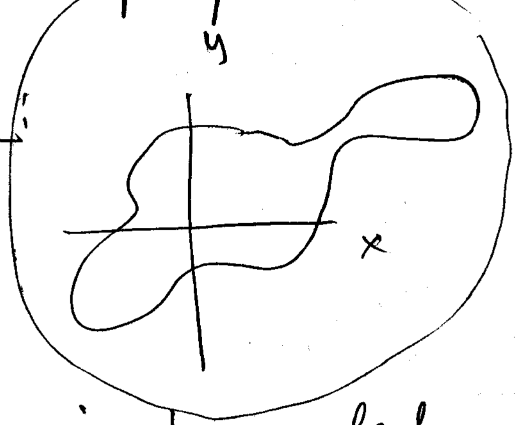
Point: If we double n (e.g. $n=4$ to start, then take $n=8$) what will that do to the error.

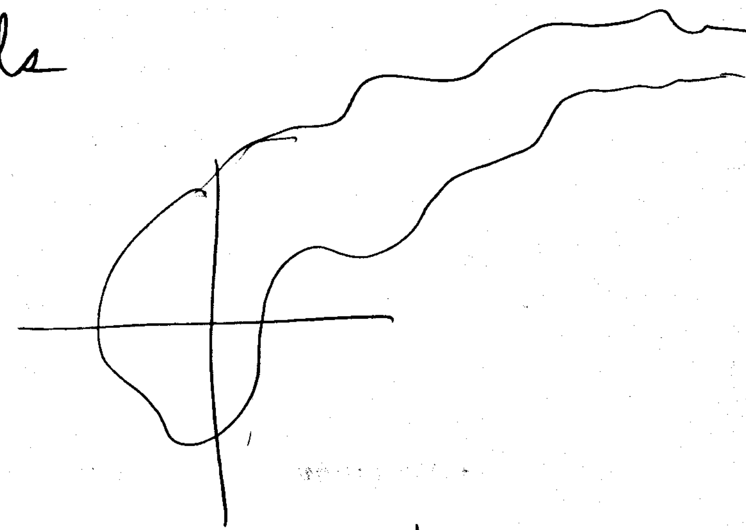
E_T and E_M will drop by about $\frac{1}{4}$

E_S will drop by about $\frac{1}{16}$

7.7 Improper Integrals

Idea:

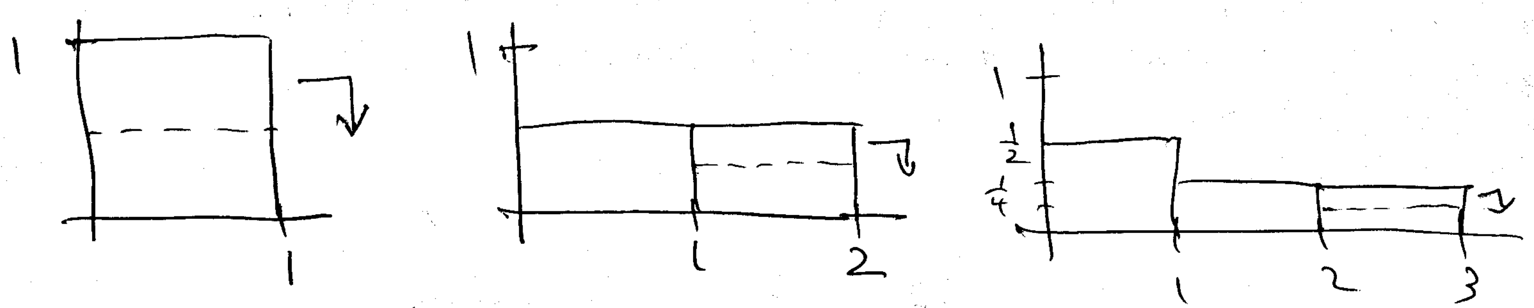


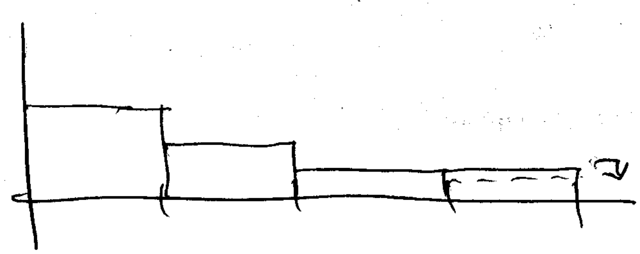


Region is bounded
Finite area

Region is unbounded
Infinite area? NOT NECESSARILY

Example:





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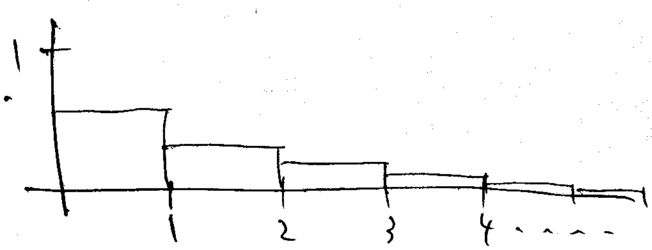
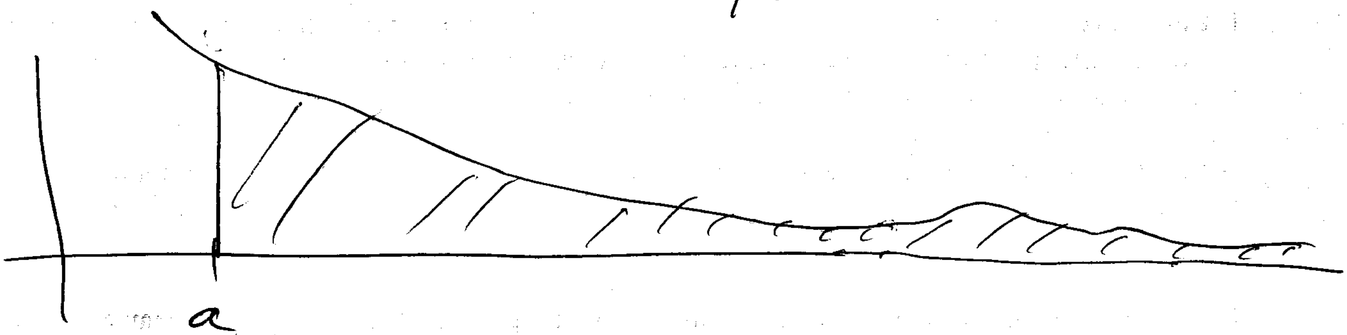


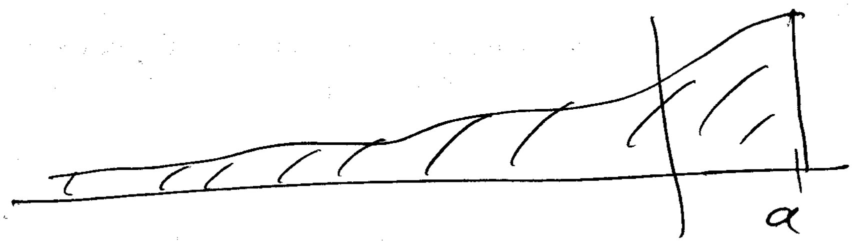
Infinite staircase has area = 1 but is not bounded.

A.

$$\int_a^{\infty} f(x)\,dx \leftarrow \text{area of unbounded region}$$



$$\int_{-\infty}^{a} f(x)\,dx$$

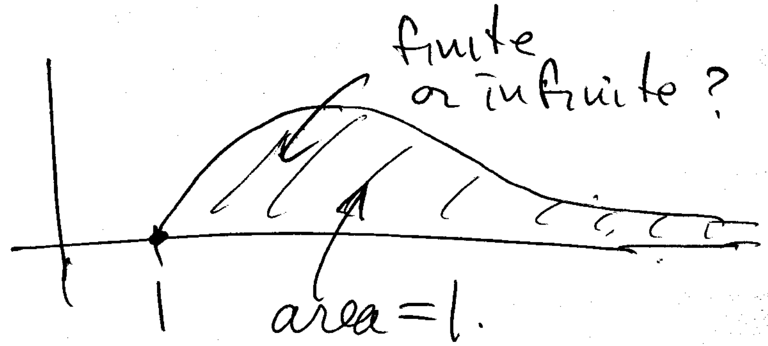


~~e.g.~~ Def:

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$$

e.g.

$$\int_1^{\infty} \frac{\ln x}{x^2}\,dx$$



$$\boxed{\lim_{x\to\infty} \frac{\ln x}{x^2} \underset{\text{L'Hopital}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = 0}$$

$$= \lim_{b\to\infty} \int_1^b \frac{\ln x}{x^2}\,dx = \lim_{b\to\infty} \underbrace{-\frac{\ln b}{b}}_{0} \underbrace{- \frac{1}{b}}_{0} + 1 = 1$$

$$\boxed{\int_1^b \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x}\Big|_1^b + \int_1^b \frac{1}{x^2}\,dx}$$

$u = \ln x \qquad dv = \frac{1}{x^2}\,dx$

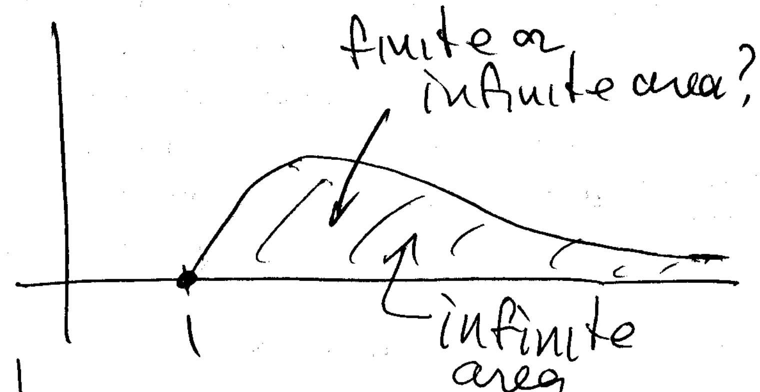
~~$dv = \frac{1}{x^2}\,dx$~~

$du = \frac{1}{x}\,dx \qquad v = \frac{-1}{x}$

$$= -\frac{\ln b}{b} + \frac{-1}{x}\Big|_1^b$$

$$= -\frac{\ln b}{b} - \frac{1}{b} + 1$$

e.g. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx$



$$\boxed{\lim_{x\to\infty} \frac{\ln x}{x} \underset{\text{L'Hopital}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0}$$

$$\int_1^\infty \frac{\ln x}{x}\,dx = \lim_{b\to\infty}\int_1^b \frac{\ln x}{x}\,dx = \lim_{b\to\infty}\frac{1}{2}(\ln b)^2 = \underline{\underline{\infty}}.$$

$$\int_1^b \frac{\ln x}{x}\,dx = \int_0^{\ln b} u\,du = \frac{1}{2}u^2\Big|_0^{\ln b} = \frac{1}{2}(\ln b)^2$$

$\cancel{u=\ln x \quad dv=\frac{1}{x}dx}$

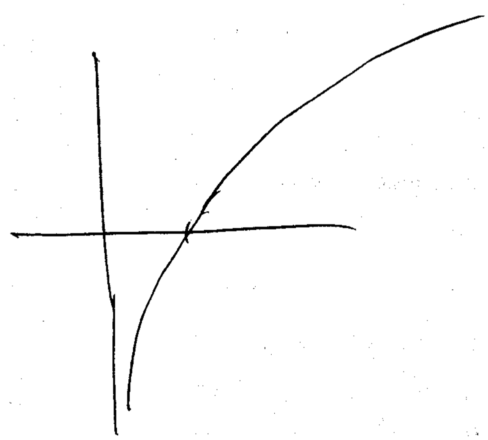
$\cancel{du=\frac{1}{x}dx \quad v=\ln x}$

$u=\ln x$

$du=\frac{1}{x}dx$

$x=1 \quad u=0$

$x=b \quad u=\ln b$



$$\int_1^\infty \frac{\ln x}{x^2}\,dx = 1$$

$$\int_1^\infty \frac{\ln x}{x}\,dx = \infty$$

$$\int_1^\infty \frac{\ln x}{x^3}\,dx = \text{finite}$$
(we expect)

$$\int_1^\infty \frac{\ln x}{x^{3/2}}\,dx = \lim_{b\to\infty}\left(-2\,\cancel{\frac{\ln b}{b^{1/2}}}^{\,0} - \cancel{\frac{4}{b^{1/2}}}^{\,0} + 4\right) = 4$$

$$\int_1^b \frac{\ln x}{x^{3/2}} \qquad u = \ln x \quad dv = x^{-3/2}\,dx$$
$$du = \frac{1}{x}\,dx \quad v = -2x^{-1/2}$$

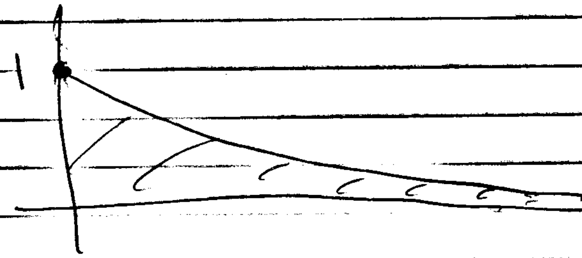
$$= -2\,\frac{\ln x}{x^{1/2}}\Big|_1^b + 2\int_1^b \frac{1}{x^{3/2}}\,dx$$

$$= -2\,\frac{\ln b}{b^{1/2}} + \left(-4\,x^{-1/2}\Big|_1^b\right)$$

$$= -2\,\frac{\ln b}{b^{1/2}} - \frac{4}{b^{1/2}} + 4$$

$$\lim_{x\to\infty}\frac{\ln x}{x^{1/2}} = \lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} 2x^{1/2}\cdot\frac{1}{x} = \lim_{x\to\infty}\frac{2}{x^{1/2}} = 0$$

$$\int_0^\infty e^{-x}\,dx$$



$$= \lim_{b\to\infty} \int_0^b e^{-x}\,dx = \lim_{b\to\infty} \left(-e^{-x} \Big|_0^b \right)$$

$$= \lim_{b\to\infty} \left(-e^{-b} + 1 \right) = 1$$

$$0$$