Trig Integrals

$$\int \sin^m(x)\cos^n(x)\,dx$$

If one of n ~~or~~ or m (or both) ~~are~~ is odd ~~then make sub~~ then this can be done.

$$\int \sin^3(x)\cos^2(x)\,dx = \int \sin^2(x)\cos^2(x)\sin(x)\,dx$$

$$= -\int (1-\cos^2(x))\cos^2(x)(-\sin(x)\,dx) \qquad u=\cos(x)$$

$$du = -\sin(x)\,dx$$

$$= -\int (1-u^2)u^2\,du$$

If both n, m are even

$$\int \sin^2(x)\cos^4(x)\,dx \longrightarrow \int \sin^2(x)\cos^3(x)\cos(x)\,dx$$

$$\sin^2(x) = \tfrac{1}{2} - \tfrac{1}{2}\cos(2x)$$

$$\cos^2(x) = \tfrac{1}{2} + \tfrac{1}{2}\cos(2x)$$

$$\cos^2(x) = 1-\sin^2(x)$$

$$\cos^3(x) = (1-\sin^2(x))^{3/2}$$

does not help.

$$= \int \left(\tfrac{1}{2}-\tfrac{1}{2}\cos(2x)\right)\left(\tfrac{1}{2}+\tfrac{1}{2}\cos(2x)\right)^2 dx$$

$$= \int \left(\tfrac{1}{2}-\tfrac{1}{2}\cos(2x)\right)\left(\tfrac{1}{4}+\tfrac{1}{2}\cos(2x)+\tfrac{1}{4}\cos^2(2x)\right) dx$$

e.g. $\int \tan(x)\,dx$

$\frac{d}{dx}\tan(x) = \sec^2(x)$

$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$

$= -\int \frac{-\sin(x)}{\cos(x)}\,dx \qquad u = \cos(x)$

$du = -\sin(x)\,dx$

$= -\int \frac{1}{u}\,du = -\ln|u| = -\ln|\cos(x)| + C.$

e.g. $\int \tan^2(x)\,dx$ $\quad \left[\int \frac{\tan^2(x)\sec^2(x)}{\sec^2(x)}\,dx \right.$ no help. $\left.\right]$

$= \int (\sec^2(x) - 1)\,dx$

$= \tan(x) - x + C$

$\frac{\sin^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$

$\tan^2(x) + 1 = \sec^2(x)$

e.g. $\int \tan^3(x)\,dx$

$= \int \frac{\sin^3(x)}{\cos^3(x)}\,dx = -\int \frac{1-\cos^2(x)}{\cos^3(x)}(-\sin(x)\,dx)$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$= -\int \frac{1-u^2}{u^3}\,du = -\int \frac{1}{u^3} - \frac{1}{u}\,du$

etc...

$\underline{\text{or}}$ $\displaystyle\int \underbrace{\tan^2(x)}\tan(x)\,dx$

$$= \int (\sec^2(x) - 1)\tan(x)\,dx$$

$$= \int \tan(x)\sec^2(x)\,dx - \int \tan(x)\,dx$$

$u = \tan x$

$du = \sec^2(x)\,dx$

$$= \int u\,du - \ln|\cos(x)| + c$$

$$= \frac{1}{2}u^2 - \ln|\cos(x)| + c$$

$$= \frac{1}{2}\tan^2(x) - \ln|\cos(x)| + c.$$

$\underline{\text{e.g.}}$ $\displaystyle\int \tan^4(x)\,dx = \int (\sec^2(x) - 1)\tan^2(x)\,dx$

$$= \int \tan^2(x)\sec^2(x)\,dx - \int \tan^2(x)\,dx$$

$u = \tan x$ $\quad du = \sec^2(x)\,dx$

e.g. $\int \sec(x)\,dx = \ln|\sec(x)+\tan(x)| + C$

$$\int \frac{\sec(x)\,(\sec(x)+\tan(x))}{\sec(x)+\tan(x)}\,dx$$

$$= \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx$$

$=$

$u = \sec(x)+\tan(x)$

$du = (\sec(x)\tan(x) + \sec^2(x))\,dx$

$$= \int \frac{1}{u}\,du \quad \text{etc---}$$

e.g. $\int \sec^3(x)\,dx = \int \sec^2(x)\,\sec(x)\,dx$

$$= \int (\tan^2(x)+1)\sec(x)\,dx$$

$$= \int \tan^2(x)\sec(x)\,dx + \int \sec(x)\,dx$$

$$= \int \frac{\sin^2(x)}{\cos^3(x)}\,dx$$

$$= \int \frac{\sin^2(x)}{\cos^4(x)}\,\cos(x)\,dx$$

$$= \int \frac{\sin^2(x)}{(1-\sin^2(x))^2}\,\cos(x)\,dx \longrightarrow \int \frac{u^2}{(1-u^2)^2}\,du$$

not doable yet.

e.g. $\int \tan^3(x)\sec(x)\,dx$

$= \int \tan^2(x)\tan(x)\sec(x)\,dx$

$= \int (\sec^2(x)-1)\sec(x)\tan(x)\,dx$

$u=\sec(x) \quad du=\sec(x)\tan(x)\,dx$

$= \int (u^2-1)\,du$ etc....

e.g. $\int \sec^4(x)\,dx = \int \sec^2(x)\sec^2(x)\,dx$

$= \int (\tan^2(x)+1)\sec^2(x)\,dx$ $\quad u=\tan(x)$, $du=\sec^2(x)\,dx$

$= \int (u^2+1)\,du$ etc...