5) $\int_0^5 \frac{5x-4}{x+1}\,dx$

$u = x+1$

$du = dx$

$x = u-1$

$5x-4 = 5(u-1)-4 = 5u-9$

$x=0 \quad u=1$

$x=5 \quad u=6$

$$= \int_1^6 \frac{5u-9}{u}\,du = \int_1^6 5 - \frac{9}{u}\,du$$

$$= 5u - 9\ln|u|\Big|_1^6 = 30 - 9\ln 6 - 5 \cancel{+0}$$

$$= 25 - 9\ln 6$$

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7) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

$u = e^x + e^{-x}$

$du = e^x - e^{-x}$

$\int \frac{du}{u} = \ln|u| + C$

$= \ln(e^x + e^{-x}) + C$

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8) $\int_0^{\pi/3} \frac{3\sin(3t)}{8-\cos(3t)}\,dt$

$u = 8 - \cos(3t)$

$du = 3\sin 3t\,dt$

$t=0 \quad u=7$

$t=\frac{\pi}{3} \quad u=9$

$$= \int_7^9 \frac{du}{u}$$

$$= \ln 9 - \ln 7$$

10) $\displaystyle\int \frac{e^{4\sqrt{s}}}{\sqrt{s}}\,ds$ $\quad u = 4\sqrt{s} = 4s^{1/2}$

$du = 2s^{-1/2}\,ds = \frac{2}{\sqrt{s}}\,ds$

$$= \frac{1}{2}\int e^{4\sqrt{s}}\left(\frac{2}{\sqrt{s}}\,ds\right) = \frac{1}{2}\int e^{u}\,du = \frac{1}{2}e^{u} + C$$

$$= \frac{1}{2}e^{4\sqrt{s}} + C$$

$\frac{d}{dx}\left(\frac{1}{6x}\right)^{4x}$ $\quad \left(\frac{1}{6x}\right)^{4x} = e^{\ln\left[\left(\frac{1}{6x}\right)^{4x}\right]} = e^{4x\ln\left(\frac{1}{6x}\right)}$

$$= e^{-4x\ln(6x)}$$

$$\frac{d}{dx} = e^{-4x\ln(6x)}\left(-\cancel{x}\cdot\frac{4x\,6}{6\cancel{x}} + (-4)\ln 6x\right)$$

$$= \left(\frac{1}{6x}\right)^{4x}\left(-4 - 4\ln 6x\right)$$

$\ln\left(\frac{1}{6x}\right) \to 6x\cdot -\frac{1}{6}x^{-2}$

$\frac{1}{6}x^{-1}$ $\quad -\frac{1}{6}x^{-2}$ $\quad = -\frac{1}{x}$

same as $-\ln(6x)$

$-\frac{1}{6x}\cdot 6 = -\frac{1}{x}$

$$\frac{d}{dx}\left(-4x\ln(6x)\right)$$

$$= (-4\cancel{x})\left(\frac{1}{\cancel{6x}}\cdot\cancel{6}\right) + \ln(6x)(-4)$$

$$= -4 - 4\ln(6x)$$

$$g(t) = 200e^{t/10}$$

$$\frac{dg}{dt} = 20e^{t/10}$$

$$\left[\ \frac{dg}{dt} = 200\left(e^{\frac{1}{10}t} \cdot \frac{1}{10}\right) = 20e^{\frac{1}{10}t}\ \right]$$

$$\frac{dg}{dt} \cdot \frac{1}{g} = \frac{20e^{t/10}}{200e^{t/10}} = \frac{1}{10}$$

$$\underline{P(t) = P_0 e^{kt}}$$

$$P(0) = P_0$$

k = growth rate if $k > 0$

decay rate if $k < 0$

We say rate is 100k percent

$$e^{.024t} = 2$$

$$.024t = \ln 2$$

$$t = \frac{\ln 2}{.024} \approx 29 \text{ yrs}$$

$$y(t) = 1300 e^{kt}$$

$$\cancel{y(25000)} = e^{25000k} = \frac{1}{3}$$

$$k = \frac{\ln(\frac{1}{3})}{25000} \quad -.000044$$

$$y(t) = 1300 e^{-.000044t}$$

$$e^{-.000044t} = \frac{1}{2} \qquad t \approx 15773$$

$$t \approx \frac{\ln(\frac{1}{2})}{-.000044}$$

$$e^{-.000044t} = \frac{1}{100} \qquad t \approx 104663$$

$$\frac{\ln(\frac{1}{100})}{-.000044\ldots}$$