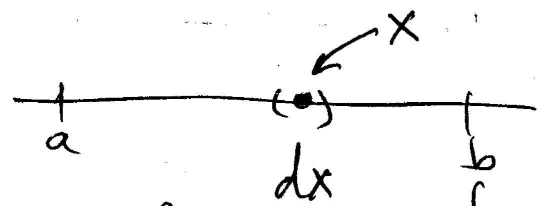
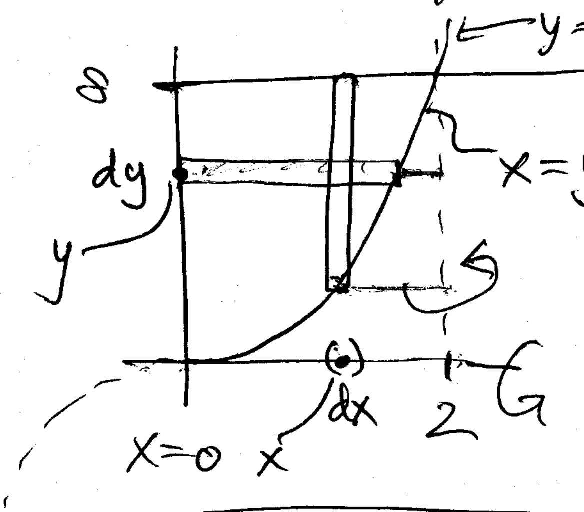
Quiz 3 6.3, 6.4

Big Idea: (1) Break up problem into smaller sub-problems by dividing an interval into small subintervals of length dx and containing a point x.



(2) Add up contributions from each dx-subinterval.

e.g. 6.4 #(14) $y = x^3$, $y = 8$, $x = 0$
revolve region around x-axis.



$y = 8$ (washers)

$$dV = \pi\left(8^2 - (x^3)^2\right) dx$$

$$= \pi(64 - x^6)\, dx$$

$$V = \int_0^2 \pi(64 - x^6)\, dx$$

(Shells) $dV = 2\pi y (y^{1/3})\, dy = 2\pi y^{4/3}\, dy$

$$V = \int_0^8 2\pi y^{4/3}\, dy$$

Rotate region about line $x=2$

(washers) $dV = \pi\left(2^2 - (2 - y^{1/3})^2\right) dy$

$= \pi\left(4 - (4 - 4y^{1/3} + y^{2/3})\right) dy$

$= \pi\left(4y^{1/3} - y^{2/3}\right) dy$

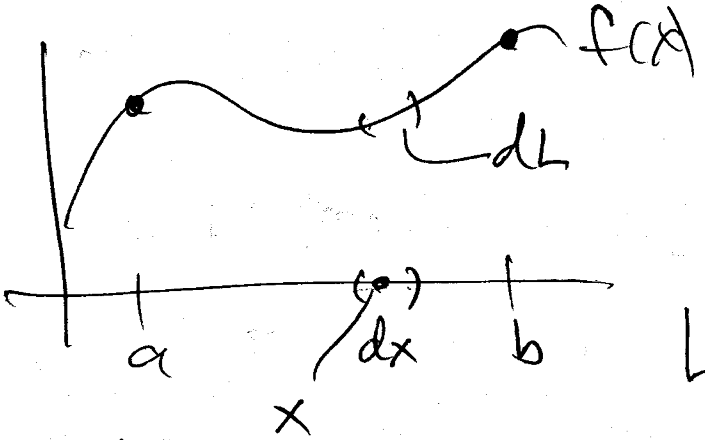
$$V = \int_0^8 \pi\left(4y^{1/3} - y^{2/3}\right) dy$$

(shells) $dV = 2\pi(2-x)(8-x^3)\,dx$

$= 2\pi(16 - 8x - 2x^3 + x^4)\,dx$

$$V = \int_0^2 2\pi(16 - 8x - 2x^3 + x^4)\,dx$$

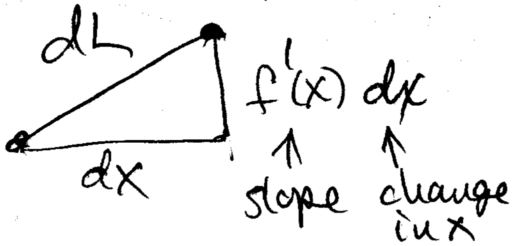
Length of curves

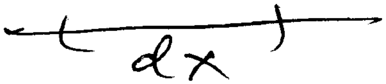


$dL = \left(dx^2 + f'(x)^2 dx^2\right)^{1/2}$

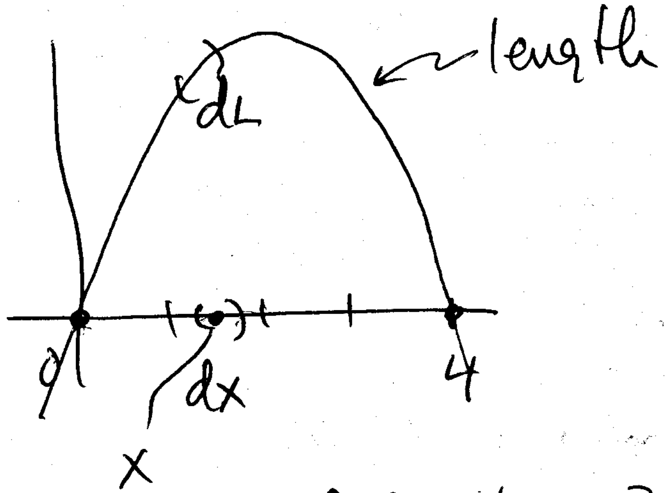
$= \left(1 + f'(x)^2\right)^{1/2} dx$

$$L = \int_a^b dL = \int_a^b \left(1 + f'(x)^2\right)^{1/2} dx$$





eg #18) $y = 4x - x^2$ on $[0, 4]$.



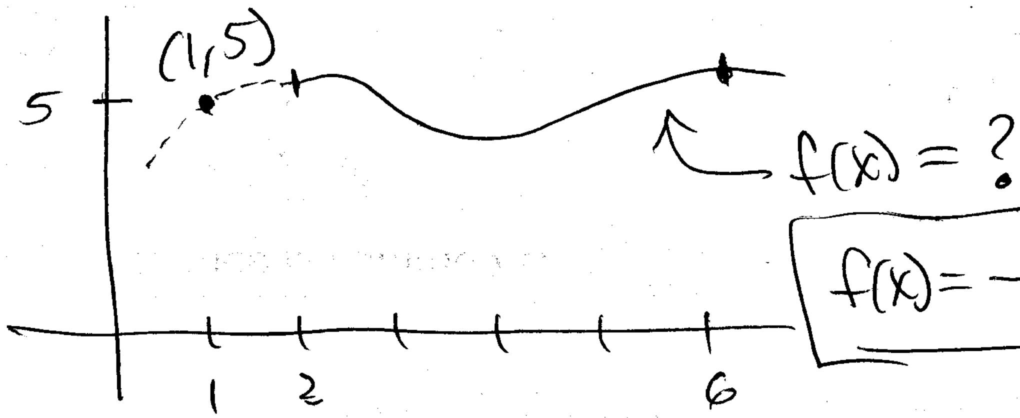
$$dL = (1 + (4-2x)^2)^{1/2}\,dx$$
$$= (1 + (16 - 16x + 4x^2))^{1/2}\,dx$$
$$= (4x^2 - 16x + 17)^{1/2}\,dx$$

$f(x) = 4x - x^2$
$f'(x) = 4 - 2x$

$$L = \int_0^4 (4x^2 - 16x + 17)^{1/2}\,dx$$

can't be done YET.

#28)



$$\boxed{f(x) = -2x^{-2} + 7}$$

$f'(x)^2 = 16x^{-6}$

$$L = \int_2^6 (1 + \underbrace{16x^{-6}}_{f'(x)^2})^{1/2}\,dx$$

$f'(x) = 4x^{-3}$

$$f(x) = \int (4x^{-3})\,dx = \frac{4x^{-2}}{-2} + C = -2x^{-2} + C$$

$f(1) = 5 \longrightarrow 5 = -2(1)^{-2} + C = -2 + C$ $\quad$ $C = 7$

6.7 Log and Exp Functions

$$\frac{d}{dx} \ln(|u(x)|) = \frac{1}{u(x)} \cdot u'(x) = \frac{u'(x)}{u(x)}$$

12) $\int \tan(10x)\,dx = \int \frac{\sin(10x)}{\cos(10x)}\,dx$

$u = \cos(10x)$

$du = -\sin(10x) \cdot 10\,dx$

$= -10\sin(10x)\,dx$

$\frac{du}{-10} = \sin(10x)\,dx$

$= \frac{-1}{10} \int \frac{-10\sin(10x)}{\cos(10x)}\,dx$

$= \frac{-1}{10} \int \frac{du}{u} = \frac{-1}{10} \int \frac{1}{u}\,du$

$= \frac{-1}{10} \ln|u| + c$

$= \frac{-1}{10} \ln|\cos(10x)| + c$ //

16) $\int \frac{e^{\sin(x)}}{\sec(x)}\,dx$

$u = \sin(x)$

$du = \cos(x)\,dx$

$\sec(x) = \frac{1}{\cos(x)}$

$\frac{1}{\sec(x)} = \cos(x).$

$= \int \cos(x)\, e^{\sin(x)}\,dx = \int e^{u}\,du = e^{u} + c$

$= e^{\sin(x)} + C.$

26) $h(t) = (\sin t)^{\sqrt{t}}$

$h'(t) = ?$ $\quad h(t) = e^{\ln([\sin t]^{\sqrt{t}})}$

$e^{\ln(\Box)} = \Box$ $\quad = e^{\sqrt{t}\ln(\sin t)}$

$\boxed{\frac{d}{dx} e^{u(x)} = e^{u(x)} \cdot u'(x)}$ $\quad h'(t) = e^{\sqrt{t}\ln(\sin t)} \left(\frac{d}{dt}\left(\sqrt{t}\ln(\sin t)\right)\right)$

$$= \underbrace{e^{\sqrt{t}\ln(\sin t)}}\left(\sqrt{t}\,\frac{1}{\sin t}\cdot\cos t + \ln(\sin t)\frac{1}{2}t^{-1/2}\right)$$

$$= (\sin t)^{(t^{1/2})}\left(t^{1/2}\cot(t) + \frac{1}{2}t^{-1/2}\ln(\sin t)\right)$$ //