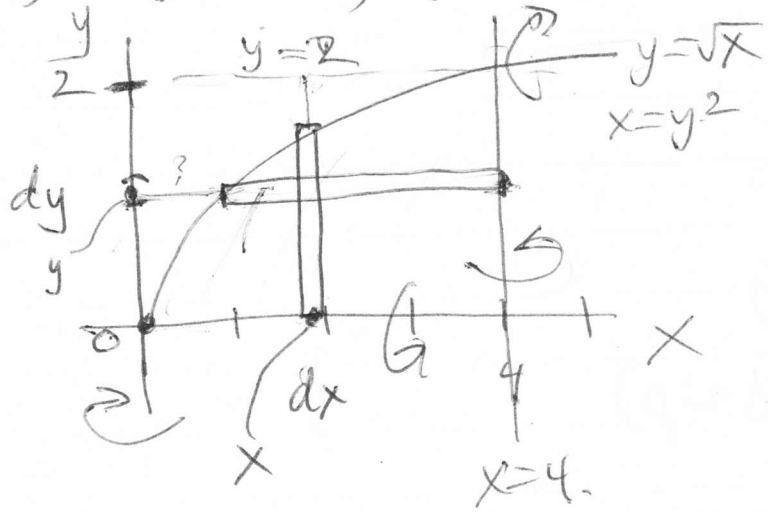
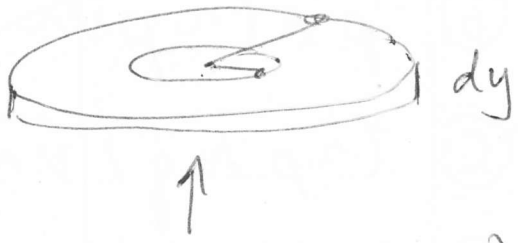
Volumes of Solids of Revolution

34) $y=\sqrt{x}$, $y=0$, $x=4$





$$dV = \pi(4^2 - (y^2)^2)\,dy$$

$$= \pi(16 - y^4)\,dy$$

$$V = \int_0^2 dV = \int_0^2 \pi(16-y^4)\,dy$$

$$= \pi\left(16y - \frac{1}{5}y^5\Big|_0^2\right)$$

$$= \pi\left(32 - \frac{32}{5} - 0\right) = \pi\left(32 - \frac{32}{5}\right) = \frac{128\pi}{5}$$

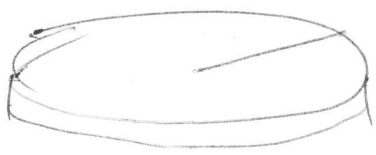
Rotate about x-axis

$$dV = \pi(\sqrt{x})^2\,dx = \pi x\,dx$$

$$V = \int_0^4 dV = \int_0^4 \pi x\,dx = \pi\left(\frac{1}{2}x^2\Big|_0^4\right)$$

$$= \pi(8-0) = 8\pi$$

Rotate about $x=4$.



$$dV = \pi(4-y^2)^2\,dy$$

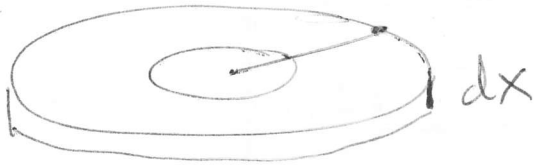
$$V = \int_0^2 dV = \int_0^2 \pi(4-y^2)^2\,dy$$

$$= \pi\int_0^2 (16-8y^2+y^4)\,dy$$

$$= \pi\left(16y - \frac{8}{3}y^3 + \frac{1}{5}y^5 \Big|_0^2\right)$$

$$= \pi\left(32 - \frac{64}{3} + \frac{32}{5}\right) = \text{etc...}$$

Rotate about $y=2$



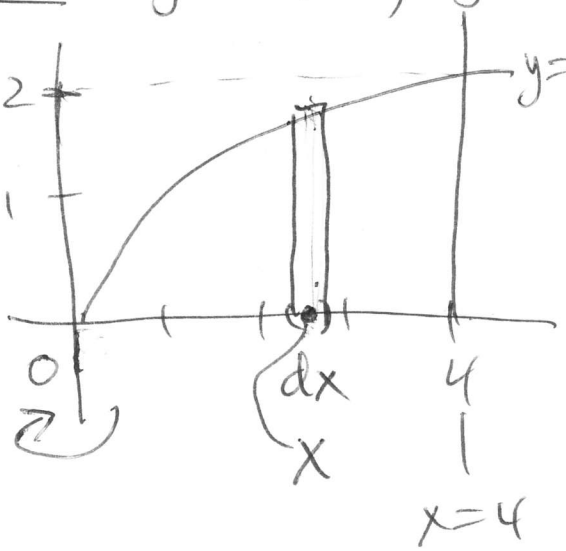
$$dV = \pi(2^2 - (2-\sqrt{x})^2)\,dx$$

$$V = \int_0^4 dV = \int_0^4 \pi(4-(4-4x^{1/2}+x))\,dx$$

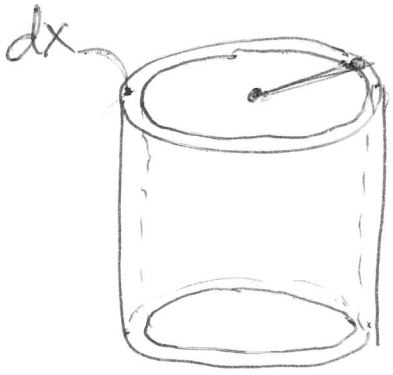
$$= \int_0^4 \pi(4x^{1/2}-x)\,dx = \cdots$$

6.4 Volume by Shells.

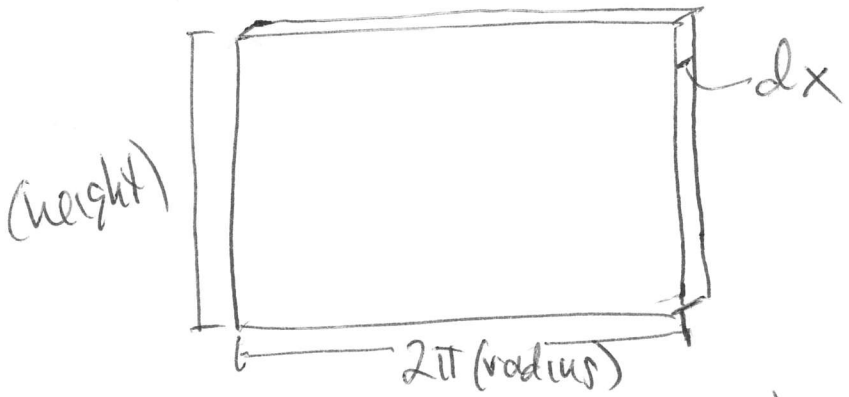
e.g. $y = \sqrt{x}$, $y = 0$, $x = 4$.



Volume by shells.



dV = volume of this shell.



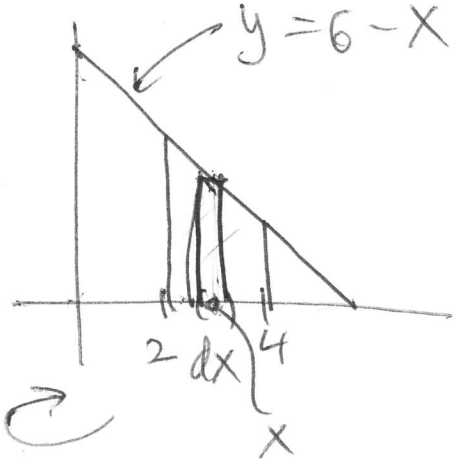
$dV = 2\pi(\text{radius})(\text{height})\,dx$

$= 2\pi x (x^{1/2})\,dx = 2\pi x^{3/2}\,dx$

$$V = \int_0^4 dV = \int_0^4 2\pi x^{3/2}\,dx = 2\pi\left(\frac{2}{5}x^{5/2}\Big|_0^4\right)$$

$$= 2\pi\left(\frac{2}{5}(4)^{5/2} - 0\right) = \frac{4\pi}{5}(32) = \frac{128\pi}{5}$$

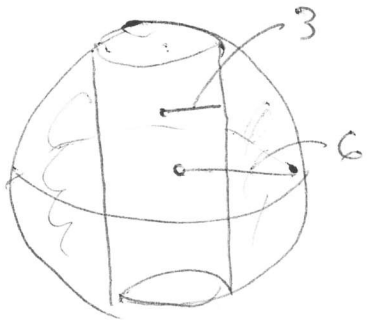
#6)

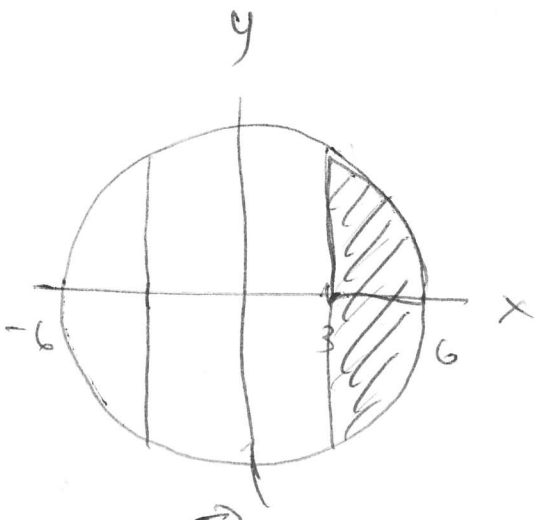


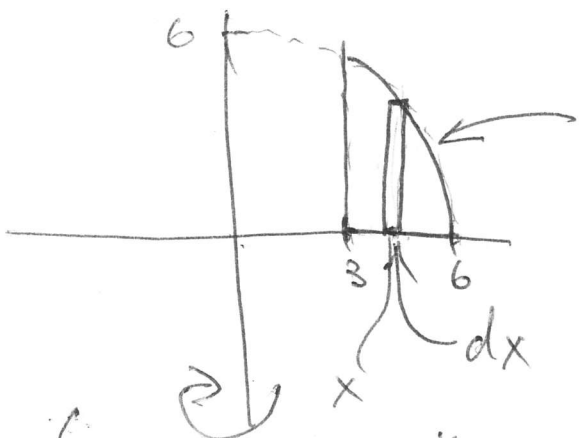
$$dV = 2\pi(x)(6-x)\,dx$$
$$= 2\pi x(6-x)\,dx$$

$$V = \int_2^4 dV = \int_2^4 2\pi x(6-x)\,dx \ \ldots$$

#24)







$x^2 + y^2 = 36$

$y = \sqrt{36 - x^2} = (36 - x^2)^{1/2}$

$dV = 2\pi x(36-x^2)^{1/2}\,dx$

$$V = -\int_3^6 (-2)\pi(x)(36-x^2)^{1/2}(dx)$$

$u = 36 - x^2$

$du = -2x\,dx$

$x = 3 \rightarrow u = 27$

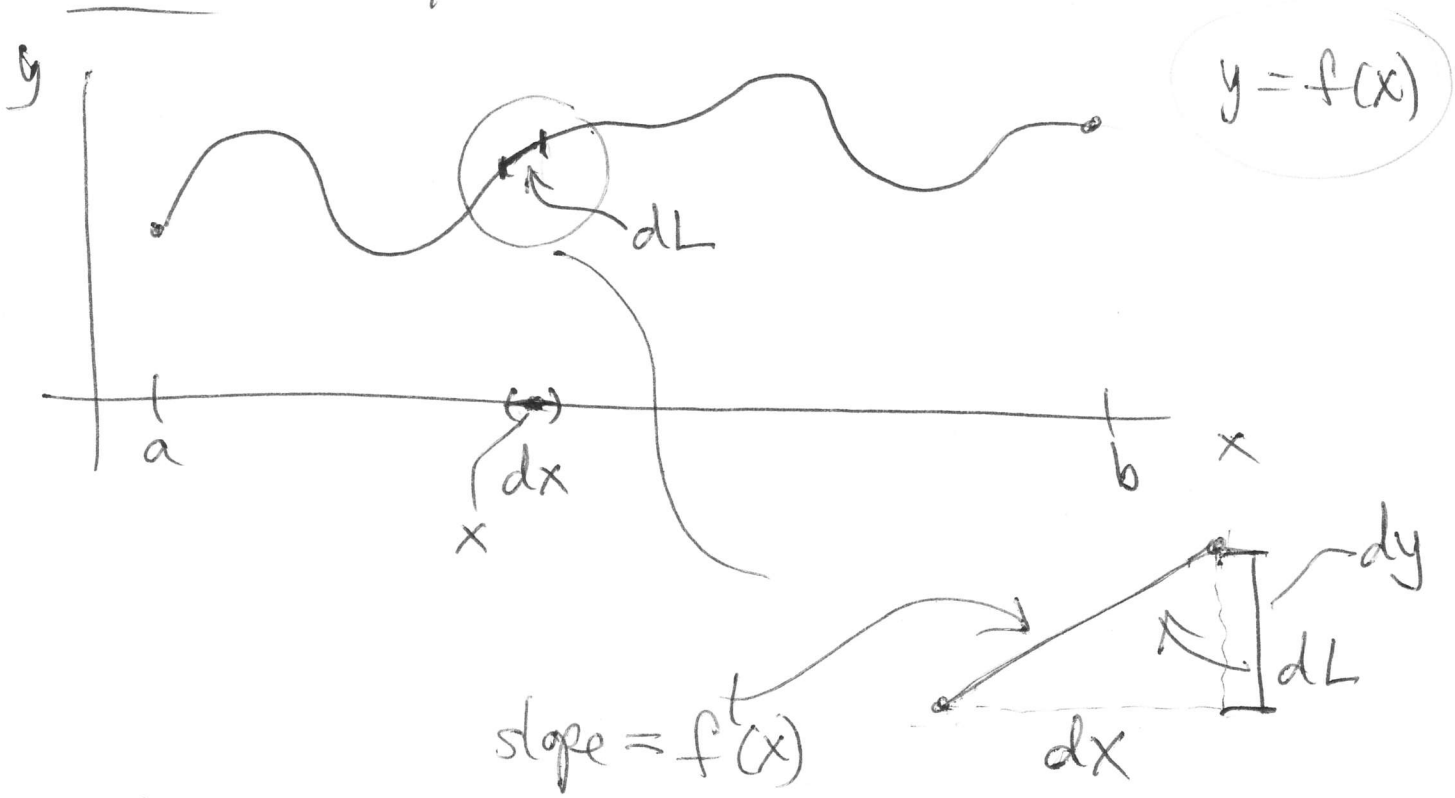
$x = 6 \rightarrow u = 0$

$$= -\int_{27}^{0} \pi u^{1/2}\,du = \int_{0}^{27} \pi u^{1/2}\,du$$

$$= \pi \frac{2}{3} u^{3/2}\Big|_0^{27} = \frac{2\pi}{3}(27^{3/2}) \;//$$

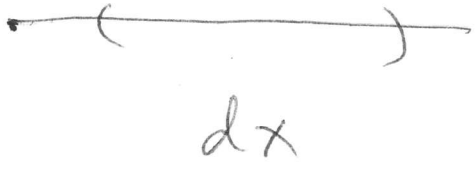
Total volume $= \frac{4\pi}{3}(27^{3/2})$ //

6.5 Length of Curves.



$$\frac{dy}{dx} = f'(x)$$

$$\boxed{dy = f'(x)\,dx}$$



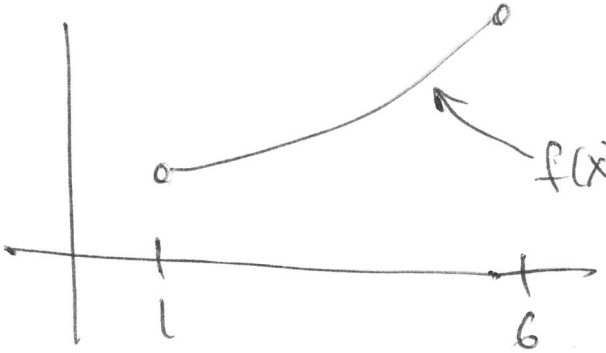
$$dL = (dx^2 + dy^2)^{1/2} = (dx^2 + [f'(x)dx]^2)^{1/2}$$

$= \left(dx^2 + (f'(x))^2 dx^2\right)^{1/2} = dx\left(1 + f'(x)^2\right)^{1/2}$

$= \left(1 + f'(x)^2\right)^{1/2} dx$

$$L = \int_a^b dL = \int_a^b \left(1 + f'(x)^2\right)^{1/2} dx \quad //$$

#6) $y = 3\ln x - \frac{x^2}{24}$ $\quad [1, 6]$



$$dL = \left(1 + f'(x)^2\right)^{1/2} dx$$

$$= \left(1 + \left(\frac{3}{x} - \frac{x}{12}\right)^2\right)^{1/2} dx$$

$$L = \int_1^6 \left(1 + \frac{9}{x^2} - \frac{1}{2} + \frac{x^2}{144}\right)^{1/2} dx$$

$$= \int_1^6 \left(\frac{9}{x^2} + \frac{1}{2} + \frac{x^2}{144}\right)^{1/2} dx$$

$\left(\frac{3}{x} + \frac{x}{12}\right)^2$

etc.....