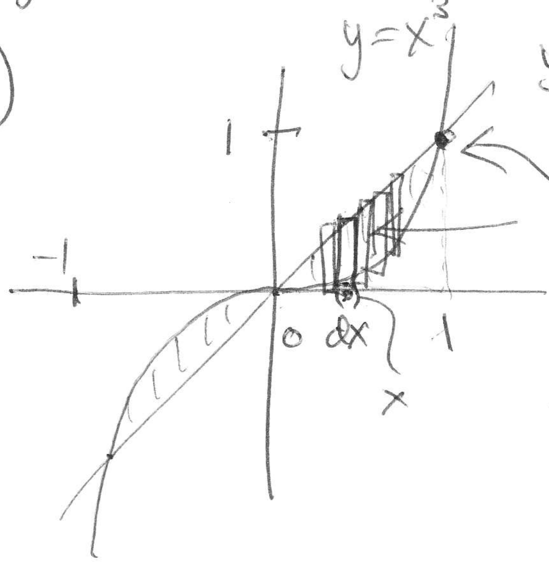
Quiz 2 - 6.1, 6.2

Regions between curves.

#6)



1. Find area of one section, then double.

$x=x^3$

$x^3-x=0$

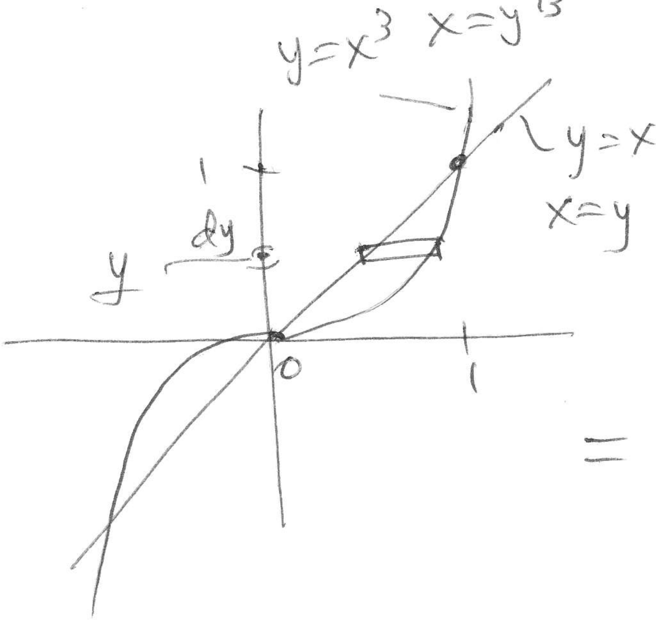
$x(x^2-1)=0$

$x=0,\ 1,\ -1$

$(1,1)$.

$$\boxed{\text{Total Area} = 2\left(\tfrac{1}{4}\right) = \tfrac{1}{2}.}$$

$$\int_0^1 (x-x^3)\,dx = \tfrac{1}{2}x^2 - \tfrac{1}{4}x^4 \Big|_0^1 = \left(\tfrac{1}{2}-\tfrac{1}{4}\right) - 0 = \tfrac{1}{4}$$

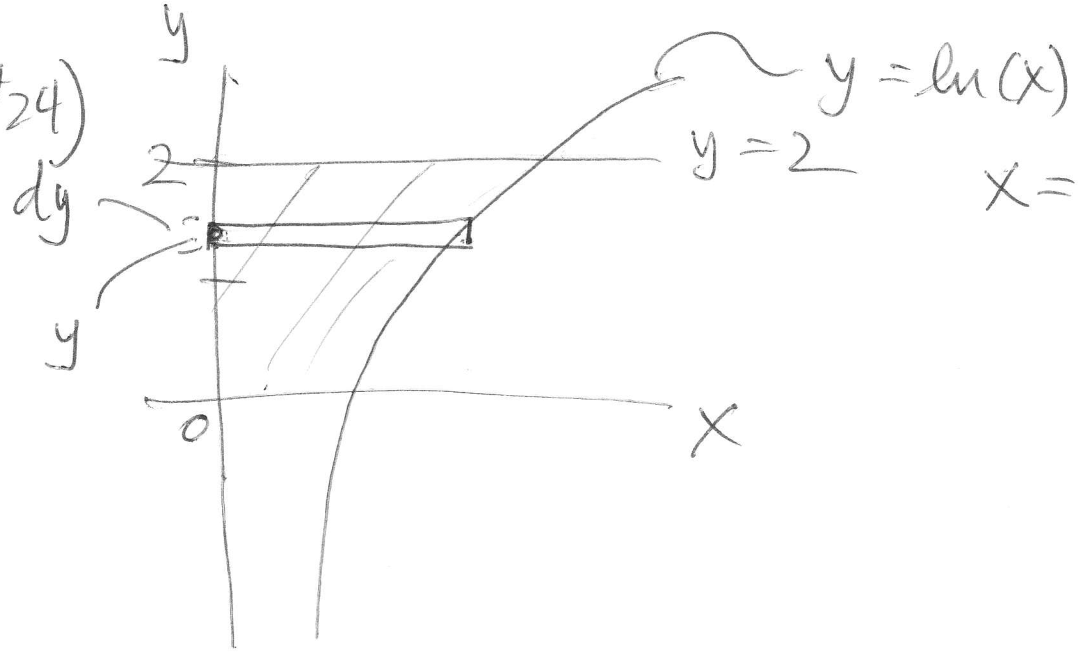


$$\int_0^1 (y^{1/3} - y)\,dy$$

$$= \tfrac{3}{4}y^{4/3} - \tfrac{1}{2}y^2 \Big|_0^1 = \tfrac{3}{4} - \tfrac{1}{2} - 0$$

$$= \tfrac{1}{4}.$$

#24)

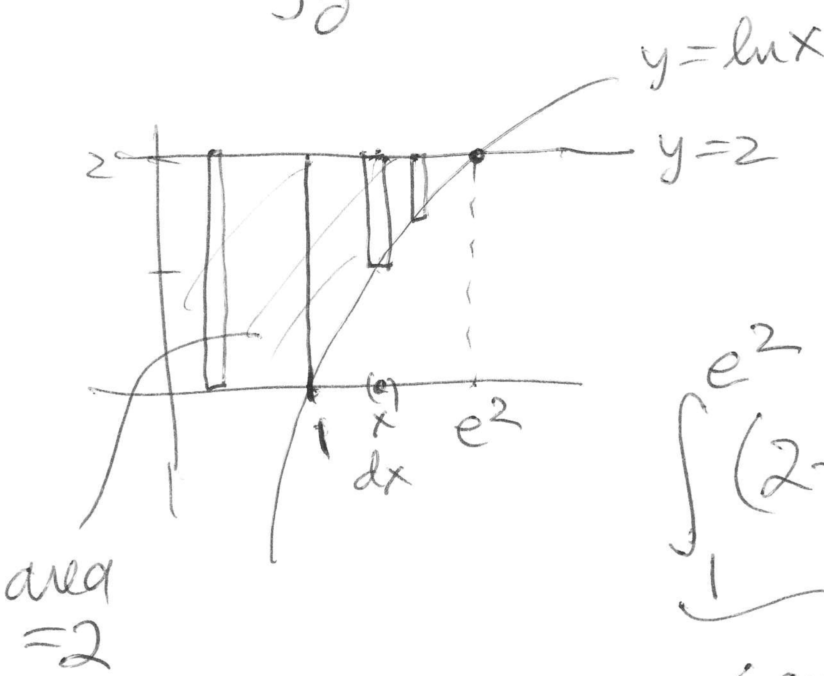


$x = e^y$

Horiz. rectangles (i.e. dy)

$$\cancel{\ln(x)\ dy}$$

$$\int_0^2 e^y\, dy = e^y\Big|_0^2 = e^2 - 1$$

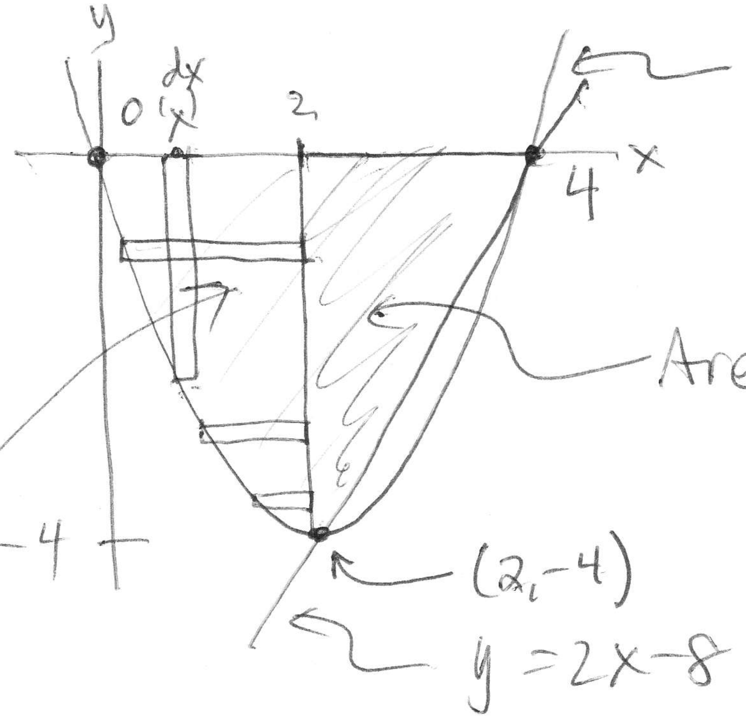


$\ln x = 2$

$x = e^2$

$$\underbrace{\int_1^{e^2} (2 - \ln x)\, dx}_{\text{can't do it now.}} + 2$$

28)



$x^2 - 4x = 0$

$x(x-4) = 0$

$\text{Area} = \frac{1}{2}(2)(4) = 4$

2 ways to get area

① vertical rectangles.

$$\int_0^2 (0 - (x^2 - 4x))\,dx = \int_0^2 4x - x^2\,dx$$

② horizontal.

$$\int_{-4}^{0} (2 - (2 + (4+y)^{1/2}))\,dy$$

$$= \int_{-4}^{0} -(4+y)^{1/2}\,dy$$

$y = x^2 - 4x$

$x^2 - 4x - y = 0$

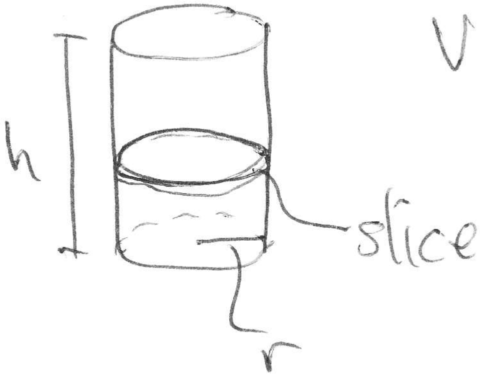
$x = \dfrac{4 \pm \sqrt{16 + 4y}}{2}$

$= \dfrac{4 \pm 2\sqrt{4+y}}{2}$

$= 2 \pm (4+y)^{1/2}$

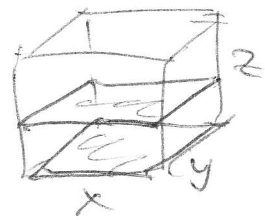
6.3 Volumes by Slicing

Idea: Cylinder

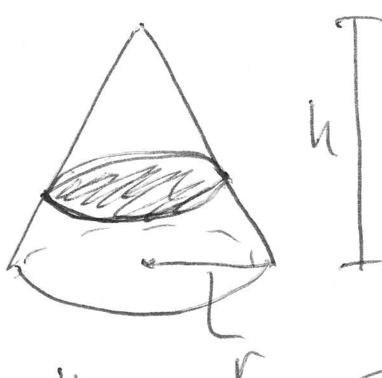


$$V = \underbrace{\pi r^2}_{\text{area of slice}} \; \underbrace{h}_{\text{height}}$$

Area of circle πr^2

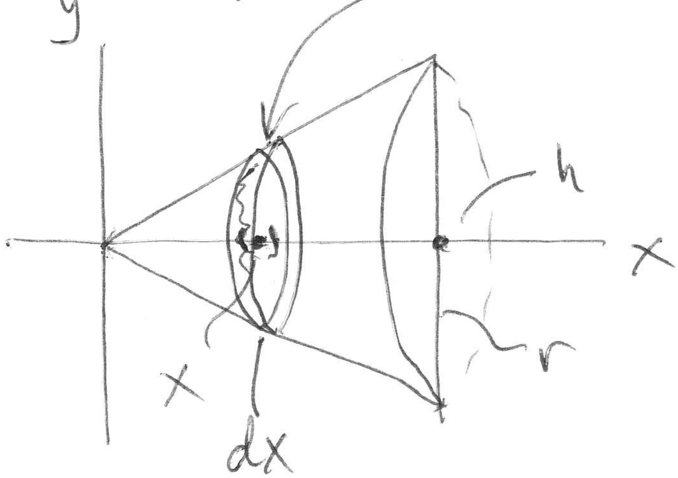


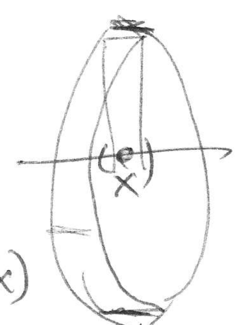
example Volume of cone.

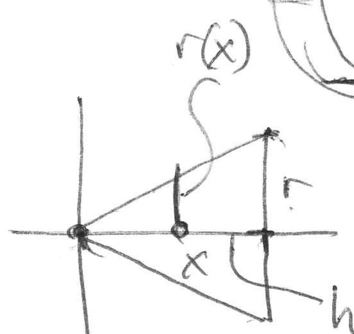


Note: areas of slices change so not so easy.

volume of disk $= \pi \big(\underbrace{\text{radius}}_{\text{depends on } x}\big)^2 dx$







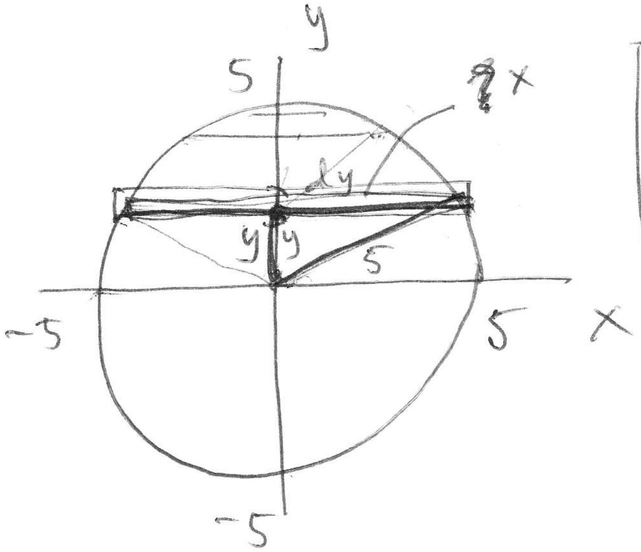
slope $= \dfrac{r}{h}$

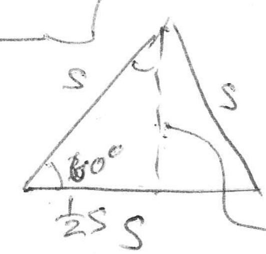
$r(x) = \dfrac{r}{h}(x)$

$$\int_0^h \underbrace{\pi\left(\frac{r}{h}x\right)^2 dx}_{\text{volume element.}} = \frac{\pi r^2}{h^2}\int_0^h x^2\,dx$$

$$= \frac{\pi r^2}{h^2}\left(\frac{1}{3}x^3\Big|_0^h\right) = \frac{\pi r^2}{\cancel{h^2}}\,\frac{1}{3}h^{\cancel{3}} = \frac{1}{3}\pi r^2 h.$$

#8)





equilateral triangle

$h = s\sin(60°)$

$$\text{Area} = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right) = \frac{\sqrt{3}}{2}s$$

$$= \frac{\sqrt{3}}{4}s^2$$

dV = volume element

$$= \frac{\sqrt{3}}{4}\left(2(25-y^2)^{1/2}\right)^2 dy$$

$$= \sqrt{3}\,(25-y^2)\,dy$$

$y^2 + x^2 = 25$

$x = \sqrt{25-y^2}$

$$V = \int_{-5}^{5}\sqrt{3}\,(25-y^2)\,dy = 2\int_0^5 \sqrt{3}\,(25-y^2)\,dy.$$

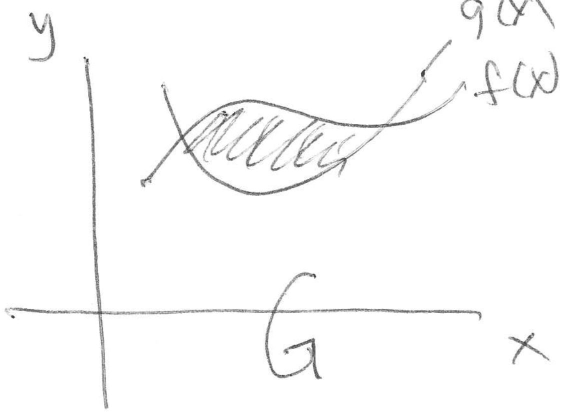
etc...

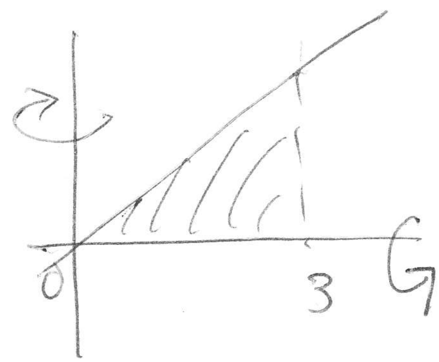
$$2\sqrt{3}\int_0^5 (25-y^2)\,dy = 2\sqrt{3}\left(25y-\frac{1}{3}y^3\Big|_0^5\right)$$

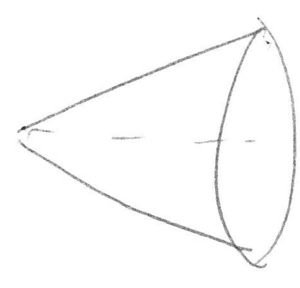
$$=2\sqrt{3}\left(125-\frac{1}{3}(125)-0\right)$$

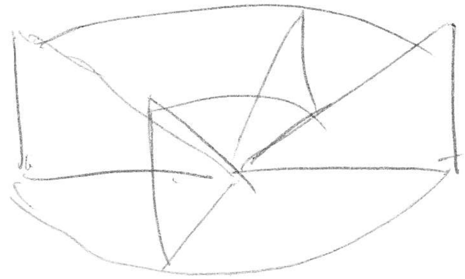
$$=2\sqrt{3}\left(\frac{2}{3}(125)\right)=\frac{4\sqrt{3}}{3}(125)=\frac{500\sqrt{3}}{3}$$

Solids of revolution

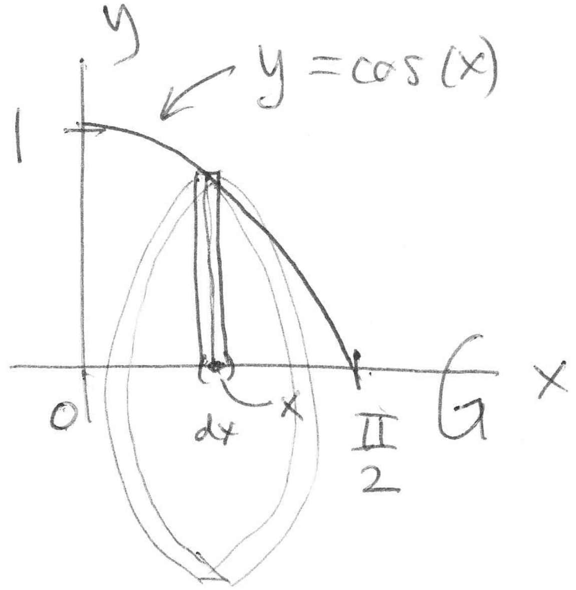


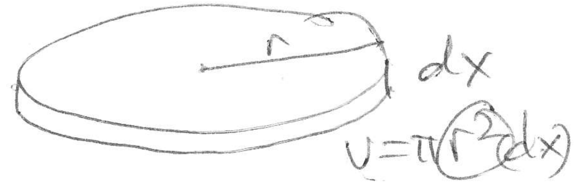






#18)





$$dV = (\pi \cos^2(x))\, dx$$

$$V = \int_0^{\pi/2} \pi \cos^2(x)\, dx$$

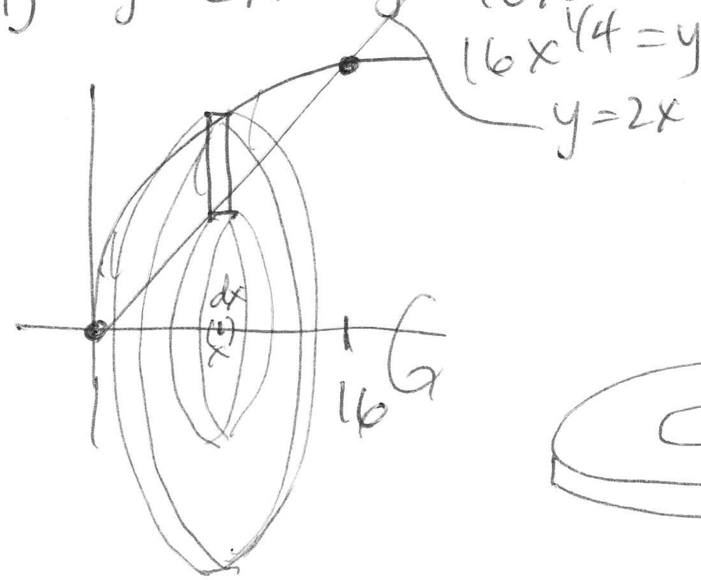
$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$= \pi \int_0^{\pi/2} \left[\frac{1}{2} + \frac{1}{2}\cos(2x)\right] dx$$

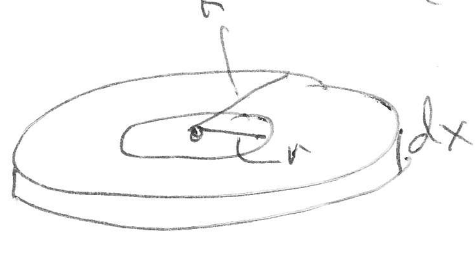
$$= \pi \left[\frac{1}{2}x + \frac{1}{4}\sin(2x) \Big|_0^{\pi/2}\right]$$

$$= \pi\left[\frac{1}{2}\cdot\frac{\pi}{2} + \frac{1}{4}\overset{0}{\cancel{\sin(\pi)}} - 0\right] = \frac{\pi^2}{4}$$

#24) $y = 2x \quad y = 16x^{1/4}$



$$2x = 16x^{1/4}$$
$$x = 8x^{1/4}$$
$$x^4 = 8^4 x$$
$$x^4 - 8^4 x = 0$$
$$x(x^3 - 8^4) = 0$$
$$x = 8^{4/3} = 16$$



$$dV = \pi (R^2)\,dx - \pi (r^2)\,dx$$

$$= \pi (R^2 - r^2)\,dx$$

$$= \pi \left((16x^{1/4})^2 - (2x)^2 \right) dx$$

$$= \pi (256x^{1/2} - 4x^2)\,dx$$

$$V = \int_0^{16} \pi (256x^{1/2} - 4x^2)\,dx$$ etc----