Displacement (or total change) vs. "distance travelled."

#14) $v(t) = \dfrac{1}{t+1}$ on $[0,8]$ $s(0) = -4$

Determine position function $s(t)$.

$$\boxed{\text{FTC: } \int_a^b f(x)\,dx = F(b) - F(a), \quad F'(x) = f(x)}$$

$$\int_0^t v(x)\,dx = s(t) - s(0)$$

$$s(t) = \int_0^t v(x)\,dx + s(0)$$

so $$s(t) = \int_0^t \frac{1}{x+1}\,dx + (-4)$$

$$= \ln|x+1| \Big|_0^t - 4$$

$$= \ln(t+1) - \cancel{\ln(1)}^{\,0} - 4$$

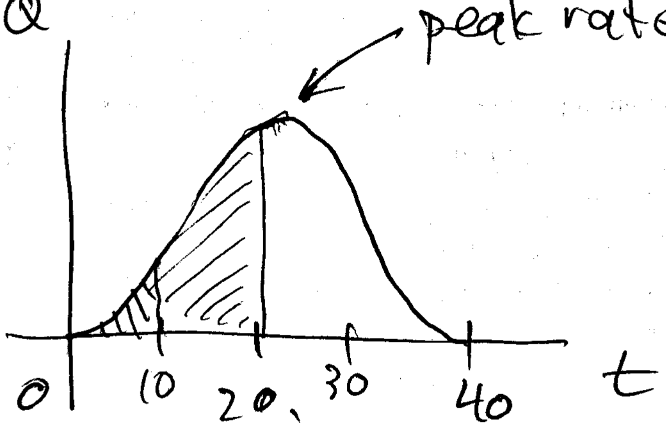
$$= \ln(t+1) - 4$$

$\int \frac{1}{x+1}\,dx$ $\quad u = x+1$, $du = dx$

$\int \frac{1}{u}\,du = \ln|u| + c$
$= \ln|x+1| + c.$

#28) $Q'(t) = 3t^2(40-t)^2$ $\frac{\text{millions of gal}}{\text{year}}$

a)




$$Q''(t) = 3t^2(2(40-t)(-1)) + 6t(40-t)^2$$
$$= (40-t)(-6t^2 + 6t(40-t))$$
$$= (40-t)(-6t^2 + 240t - 6t^2)$$
$$= -6t(40-t)(2t-40)$$

crit pts: $t=0$, $t=40$, $\underline{t=20}$

b) How much oil extracted is 1st 10 years?

$$\int_0^{10} Q'(t)\,dt = Q(10) - Q(0)$$
$$\int_0^{10} 3t^2(40-t)^2\,dt = \int_0^{10} 3t^2(1600 - 80t + t^2)\,dt$$
$$= \int_0^{10} 4800t^2 - 240t^3 + 3t^4\,dt$$
$$= \frac{4800}{3}t^3 - \frac{240}{4}t^4 + \frac{3}{5}t^5 \Big|_0^{10} = 1600t^3 - 60t^4 + \frac{3}{5}t^5 \Big|_0^{10}$$

$1{,}600{,}000 - 600{,}000 + 60000 - 0 = 1{,}060{,}000$ mil gal.

20 years? $\int_0^{20} 3t^2(40-t)^2\,dt$

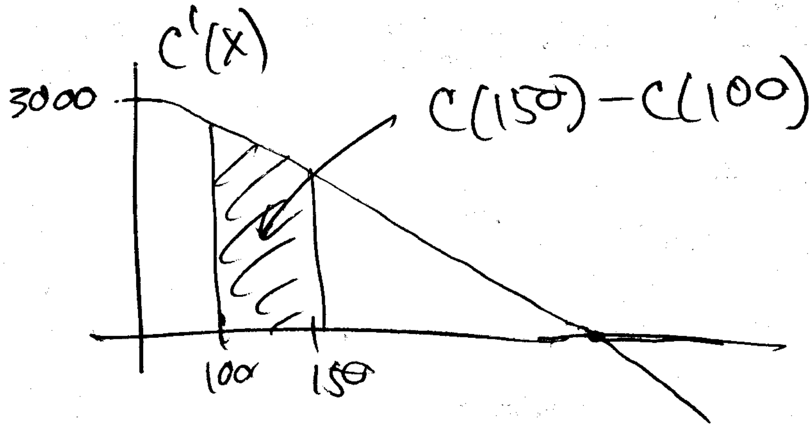
between 10 and 20? $\int_{10}^{20} 3t^2(40-t)^2\,dt$

etc...

#38) $C(x)$ = cost function $\rightarrow$ total cost of producing x units of whatever.

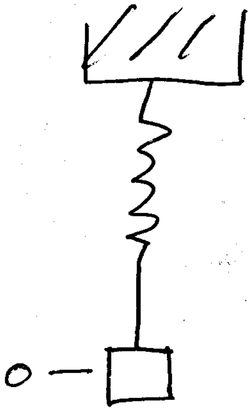
$C'(x)$ = marginal cost $\rightarrow$ cost per unit when making x units.

$C'(x) = 3000 - x - .001x^2$




a) $\int_{100}^{150} C'(x)\,dx = C(150) - C(100)$

15) $v(t) = 2\pi \cos \pi t, \quad t \geq 0, \quad s(0) = 0.$



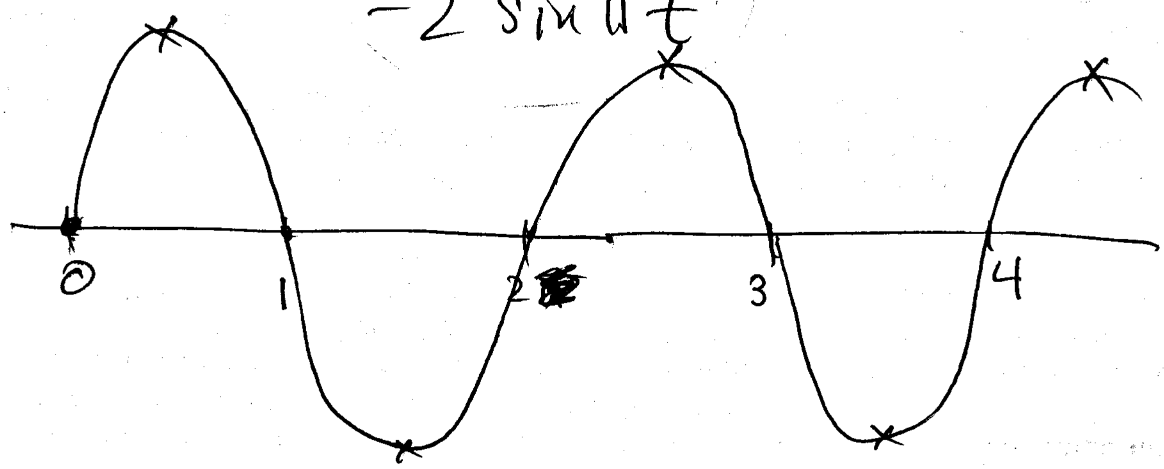
(a) $s(t) = \int_0^t 2\pi \cos(\pi x)\,dx + s(0)$

$= 2\pi \left( \frac{1}{\pi} \sin(\pi x) \Big|_0^t \right) + 0$

$= 2 \sin \pi t - 2 \cancel{\sin(\pi \cdot 0)}^{0}$

$= 2 \sin \pi t$

(c)




25) $a(t) = 88\ \mathrm{ft/s^2}$

(a) $v(t) = \int_0^t a(x)\,dx + v(0)$

$= \int_0^t 88\,dx + 0 = 88t$

$s(t) = \int_0^t v(x)\,dx + s(0)$

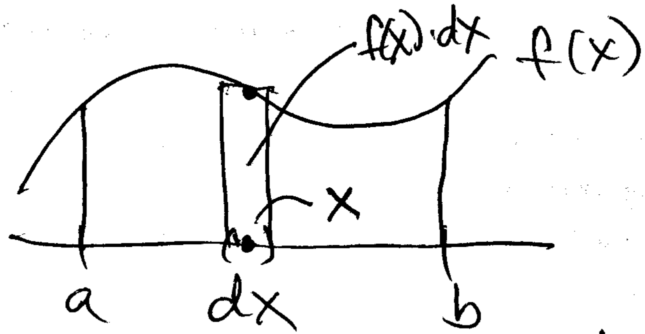
$= \int_0^t 88x\,dx + 0 = 44t^2$

(b) $s(4) - s(0)$ (c) $44t^2 = \frac{5280}{4} = 1320$ ....

# 6.2 Regions Between Curves.

More geometric point of view, i.e., think "area under curve":

$$\int_a^b \underbrace{f(x)\,dx}_{\text{area of infinitesimal rectangle.}}$$

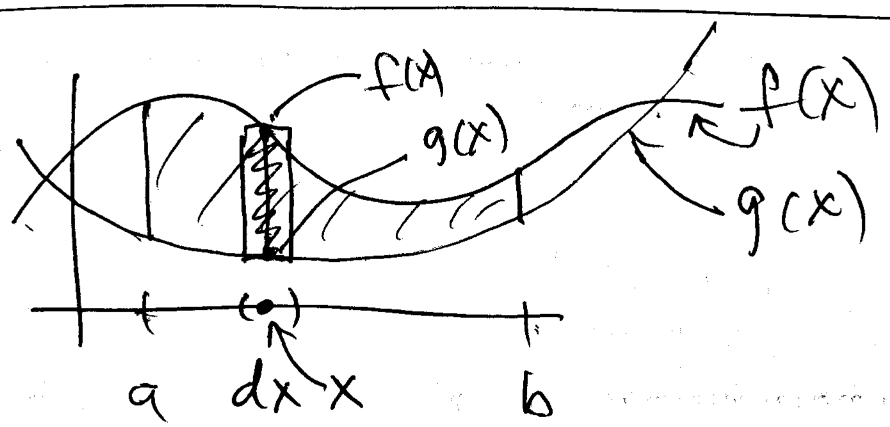



Idea: ① Divide [a,b] into small subintervals
② Find area over each subinterval.
③ Add them all up.

$$\int_a^b \underbrace{f(x)\,dx}_{\text{area of small rectangle}}$$

sum up small areas

Idea: Usually sufficient to focus on one typical subinterval.

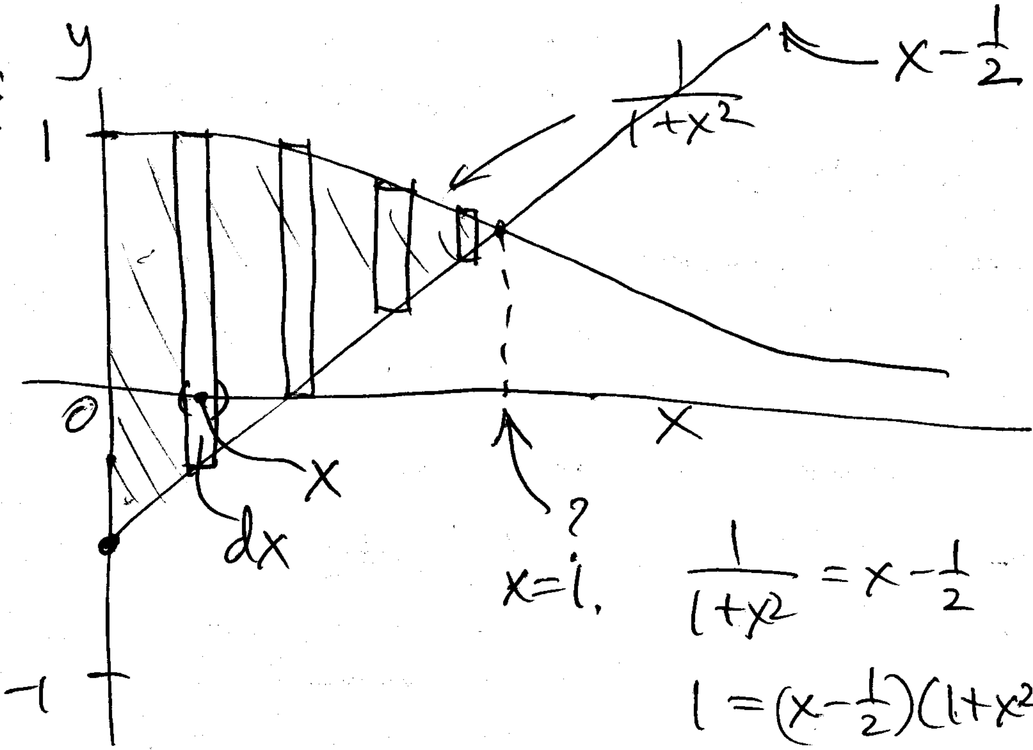



Area of small rectangle is: $[f(x)-g(x)]\,dx$

Area: $\int_a^b [f(x)-g(x)]\,dx$.

e.g. 1 $f(x)=\dfrac{1}{1+x^2}$, $g(x)=x-\frac{1}{2}$, y-axis

Sketch:




$$\frac{1}{1+x^2}=x-\frac{1}{2}$$

$$1=(x-\tfrac{1}{2})(1+x^2)$$

$$1=x^3+x-\tfrac{1}{2}x^2-\tfrac{1}{2}$$

$$x^3+x-\tfrac{1}{2}x^2=\tfrac{3}{2}.\ \text{hard}$$

Area element $=\left(\dfrac{1}{1+x^2}-(x-\tfrac{1}{2})\right)dx$

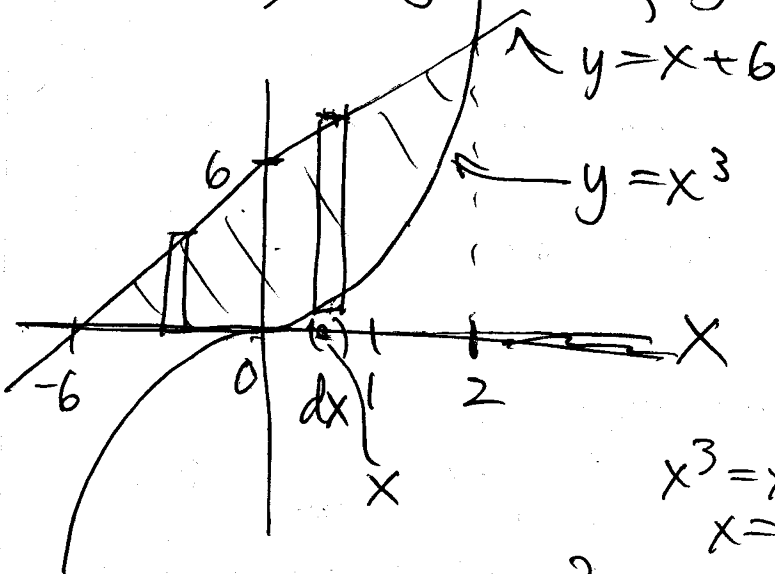
Area $\displaystyle\int_0^1\left(\frac{1}{1+x^2}-x+\frac{1}{2}\right)dx$

$$=\tan^{-1}(x)-\tfrac{1}{2}x^2+\tfrac{1}{2}x\Big|_0^1$$

$$=\left(\tan^{-1}(1)-\tfrac{1}{2}+\tfrac{1}{2}\right)-\left(\tan^{-1}(0)-0+0\right)$$

$$=\frac{\pi}{4}$$

eg. 3) $y = x^3$, $y = x+6$, x-axis.

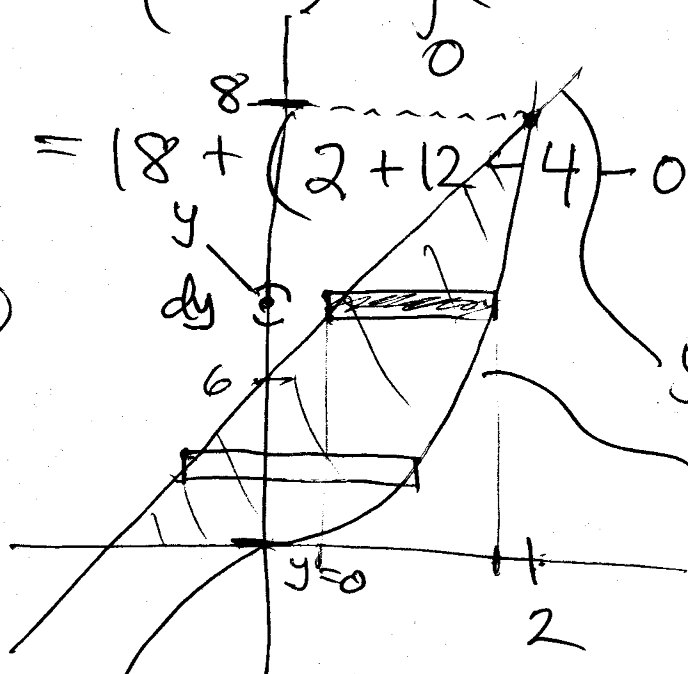



$x^3 = x+6$

$x = 2$

① $A(\triangle) + \int_0^2 (x+6-x^3)\,dx = \frac{1}{2}(6)(6) + \left(\frac{1}{2}x^2 + 6x - \frac{1}{4}x^4\right)\Big|_0^2$

$= 18 + (2 + 12 - 4 - 0) = 28$ //

② 


$y = x+6 \rightarrow x = y-6$

$y = x^3 \rightarrow x = y^{1/3}$

$$A = \int_0^8 \left[y^{1/3} - (y-6)\right] dy = \frac{3}{4}y^{4/3} - \frac{1}{2}y^2 + 6y \Big|_0^8$$

$= \frac{3}{4}(8)^{4/3} - \frac{1}{2}\cdot 64 + 48 - 0$

$= 12 - 32 + 48 = 28$ //