MATH 113 – 27 SEPTEMBER 2005 – EXAM 1

Answer each of the following questions. Show all work, as partial credit may be given.

1. (3 pt. each) Consider the function $f(x)$ whose graph is sketched below. Determine whether each of the following statements is true or false (CIRCLE ONE).

\[
\lim_{x \to 2} f(x) = 2. \quad \text{T} \quad \text{F} \quad \lim_{x \to 2} f(x) = f(2). \quad \text{T} \quad \text{F}
\]

$f(x)$ is continuous at $x = 2$. \quad \text{T} \quad \text{F} \quad \lim_{x \to 1^-} f(x)$ exists. \quad \text{T} \quad \text{F}

\[
\lim_{x \to 1^+} f(x) \text{ exists.} \quad \text{T} \quad \text{F} \quad \lim_{x \to 1} f(x) \text{ exists.} \quad \text{T} \quad \text{F}
\]

$f(x)$ is continuous at $x = 0$. \quad \text{T} \quad \text{F} \quad f(x)$ is differentiable at $x = 0$. \quad \text{T} \quad \text{F}

\[
\lim_{x \to 3^-} f(x) = 0. \quad \text{T} \quad \text{F} \quad \lim_{x \to 3^+} f(x) = 1. \quad \text{T} \quad \text{F}
\]

\[
\lim_{x \to -2^+} f(x) = +\infty. \quad \text{T} \quad \text{F} \quad \lim_{x \to -2^-} f(x) = +\infty. \quad \text{T} \quad \text{F}
\]

2. (8 pts. each) Evaluate each of the following limits.

(a) \[
\lim_{y \to 0} \frac{y}{\sin(3y)}
\]

(b) \[
\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1}
\]
3. (8 pts. each) Let \( f(x) = \frac{x^2 - 3x}{x^2 - 4} \). Evaluate each of the following limits. If the limit is infinite you must determine whether or not it is \(+\infty\) or \(-\infty\).

   (a) Find \( \lim_{x \to -\infty} f(x) \)

   (b) Find \( \lim_{x \to 2^-} f(x) \)

   (c) Find \( \lim_{x \to 0} f(x) \)

4. (12 pts.) Find the slope of the tangent line to the graph of the function \( f(x) = \frac{x}{x - 2} \) at the point \((3, 3)\) by calculating the limit of the difference quotient. Then find an equation for the line tangent to the graph there.

5. (12 pts.) Find the derivative \( f'(x) \) of the function \( f(x) = \sqrt{1 + x} \) by using the definition of the derivative as the limit of the difference quotient.