Quiz 12 - Tuesday 12/11

Final Exam - Wed 12/12

Monday 12/10 - Informal review.

Tuesday recitations are <u>on</u>

Substitution rule.

e.g. $\int x \sin(2x^2)\,dx$ $\quad u = 2x^2$

$\frac{du}{dx} = 4x \rightarrow du = 4x\,dx$

$= \frac{1}{4}\int \sin(2x^2)(4x\,dx)$

$= \frac{1}{4}\int \sin(u)\,du = \frac{1}{4}(-\cos(u)) + c$

$= \frac{-1}{4}\cos(2x^2) + c$

e.g. $\int \frac{\ln(t)}{t}\,dt$ $\quad u = \frac{1}{t}$

$du = \frac{-1}{t^2}\,dt$

$u = \frac{1}{t}$

$t = \frac{1}{u}$

$du = \frac{-1}{t}\left(\frac{1}{t}\,dt\right)$

$= -u\left(\frac{1}{t}\,dt\right)$

$= \int \frac{\ln\left(\frac{1}{u}\right)}{1} \cdot \frac{-du}{u}$ $\quad \frac{1}{t}\,dt = -\frac{du}{u}$

$$\boxed{\ln\left(\frac{1}{u}\right) = \ln(u^{-1}) = -\ln(u)}$$

$$= \int -\ln(u) \cdot \frac{-du}{u} = \int \frac{\ln(u)}{u}\, du$$

Substitution is correct but did not help.

Here is another attempt.

$$\int \frac{\ln(t)}{t}\, dt \quad du \qquad\qquad u = \ln(t)$$
$$du = \frac{1}{t}\, dt$$

$$= \int u\, du = \frac{1}{2}u^2 + c = \frac{1}{2}(\ln(t))^2 + c.$$

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eg $\int x e^{x^2} dx \qquad u = x^2$
$\qquad\qquad\qquad du = 2x\, dx$

$$= \frac{1}{2}\int e^{x^2}(2x\, dx) = \frac{1}{2}\int e^u\, du = \frac{1}{2}e^u + c$$

$$= \frac{1}{2}e^{x^2} + c$$

eg $\displaystyle\int \frac{1}{10x-3}\,dx$

$u = 10x-3$

$du = 10\,dx$

$$= \frac{1}{10}\int \frac{1}{10x-3}(10\,dx)$$

$$= \frac{1}{10}\int \frac{1}{u}\,du = \frac{1}{10}\ln|u| + C$$

$$= \frac{1}{10}\ln|10x-3| + C$$

e.g. $\displaystyle\int \frac{x}{(x-4)^{1/2}}\,dx$

$u = \dfrac{1}{(x-4)^{1/2}} = (x-4)^{-1/2}$

$du = -\frac{1}{2}(x-4)^{-3/2}\,dx$

$$= \int x(x-4)^{-1/2}\,dx$$

$$= -\frac{1}{2}\underbrace{\frac{1}{(x-4)}}_{u^2}\cdot\underbrace{\frac{1}{(x-4)^{1/2}}}_{u}\,dx$$

$$= -\frac{1}{2}u^3\,dx$$

$$dx = -2\frac{du}{u^3}$$

ok stop here.

$u = x-4 \rightarrow x = u+4$

$du = dx$

$$= \int \frac{u+4}{u^{1/2}}\,du \rightarrow \frac{u}{u^{1/2}} + \frac{4}{u^{1/2}}$$

$$= \int \left(u^{1/2} + 4u^{-1/2}\right)du$$

$$= \frac{2}{3}u^{3/2} + 8u^{1/2} + C$$

$$= \frac{2}{3}(x-4)^{3/2} + 8(x-4)^{1/2} + C.$$

$$\int_0^4 \frac{p}{(9+p^2)^{1/2}}\,dp \qquad \begin{aligned} u &= 9+p^2 \\ du &= 2p\,dp \end{aligned}$$

$p=0 \rightarrow u=9$

$p=4 \rightarrow u=25$

Only difference: change my limits of integration to new variable

$$= \frac{1}{2}\int_0^4 \frac{1}{(9+p^2)^{1/2}}(2p\,dp)$$

$$= \frac{1}{2}\int_9^{25} \frac{1}{u^{1/2}}\,du = \frac{1}{2}\int_9^{25} u^{-1/2}\,du$$

$$= \frac{1}{\cancel{2}}\left(\cancel{2}\,u^{1/2}\right)\Big|_9^{25} = (25)^{1/2} - (9)^{1/2} = 5-3 = 2$$

$$\int_0^{\pi/4} \frac{\sin x}{\cos^3 x}\,dx \qquad \begin{aligned} u &= \cos x \\ du &= -\sin x\,dx \end{aligned}$$

$x=0 \longrightarrow u=1$

$x=\frac{\pi}{4} \rightarrow u=\frac{\sqrt{2}}{2}$

$$= -\int_0^{\pi/4} \frac{1}{\cos^3 x}(-\sin x\,dx)$$

$$= -\int_{1}^{\frac{\sqrt{2}}{2}} \frac{1}{u^3}\,du = -\int_{1}^{\sqrt{2}/2} u^{-3}\,du$$

$$= +\left(+\frac{1}{2}u^{-2}\Big|_{1}^{\frac{\sqrt{2}}{2}}\right) = \frac{1}{2}\left(\left(\frac{\sqrt{2}}{2}\right)^{-2} - (1)^{-2}\right)$$

$$\hookrightarrow \left(\frac{2}{\sqrt{2}}\right)^2 = \frac{4}{2} = 2$$

$$= \frac{1}{2}(2-1) = \frac{1}{2}$$

$$\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{1}{\frac{2}{4}} = \frac{4}{2} = 2$$