Exam 3 returned in Recitation (I hope)
Quiz 11 (new) 5.1, 5.2
Quiz 12 (12-11) 5.3, 5.4 ← There will be a recitation on Dec. 11.
Final Exam: Dec 12 $1^{30}-4^{15}$.
- About 60% of final will cover 5.1-5.5
- Remainder will cover all other chapters.
- No calculators.
- Study backwards, i.e. Ch 5, Ch 4, Ch 3, Ch 2,
- I will give a list of sections covered.

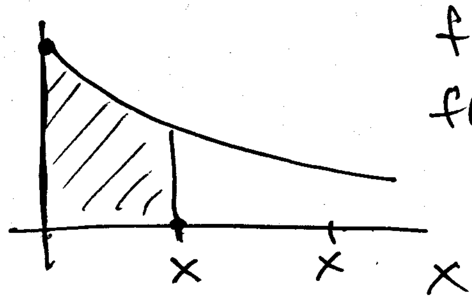
FTC: (I) $$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

(II) $$\int_a^b f(x)\,dx = F(b) - F(a)$$

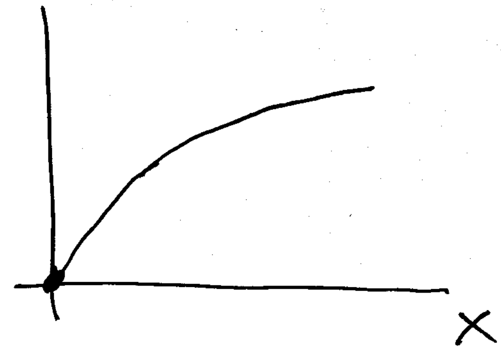
where $F'(x) = f(x)$.

e.g. #73, p 348)

(a)



$f(x) > 0$
$f(x)$ decr



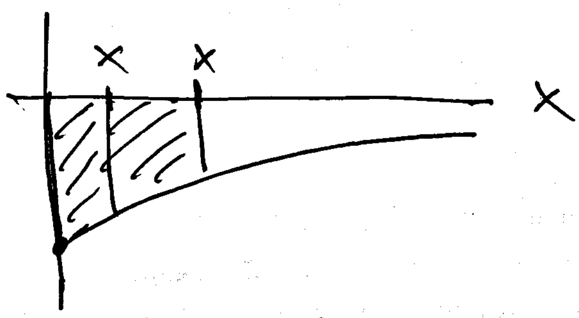
$y = f(x)$

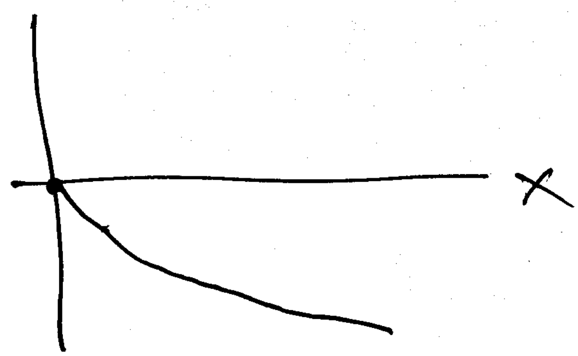
$y = A(x)$

$$A(x) = \int_0^x f(t)\,dt.$$

As long as $f(x) > 0$, $A(x)$ will be increasing

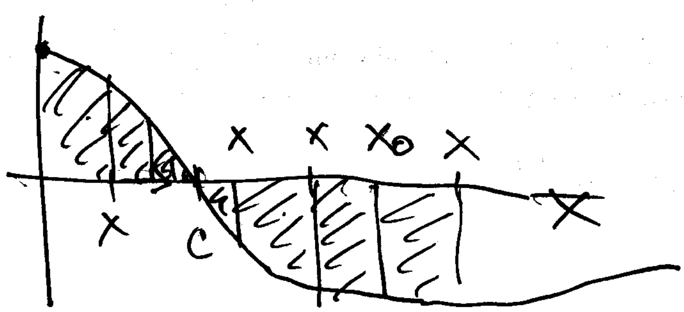
(b)

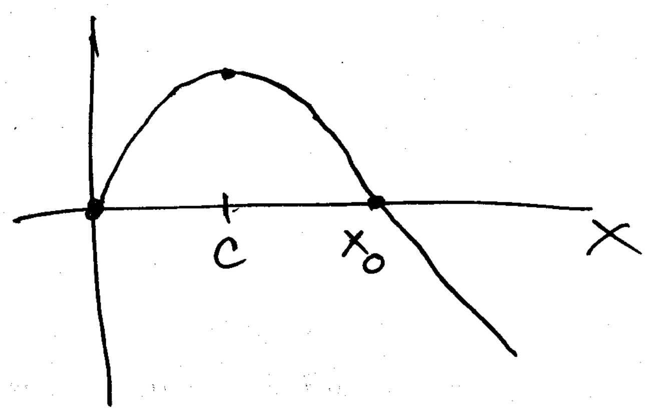




$y = f(x)$

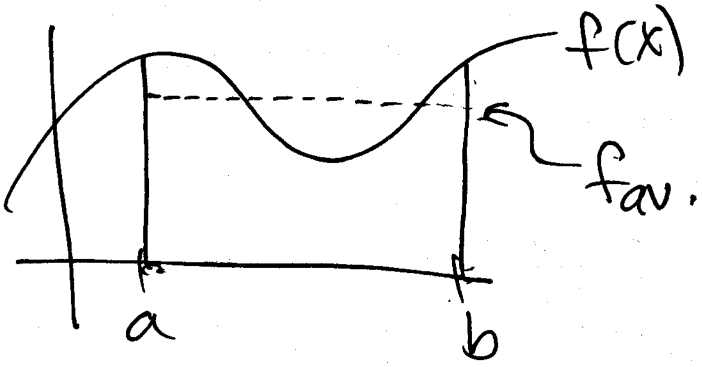
$y = A(x)$





5.4 Working with Integrals.

Average value

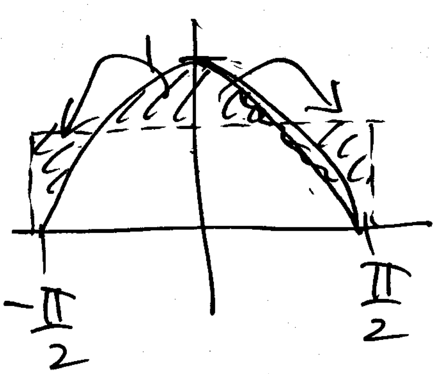


what is the average value of f on [a,b]?

want $(f_{av})(b-a) = \int_a^b f(x)\,dx$

$$f_{av} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

eg 21) $f(x) = \cos(x)$ $\quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



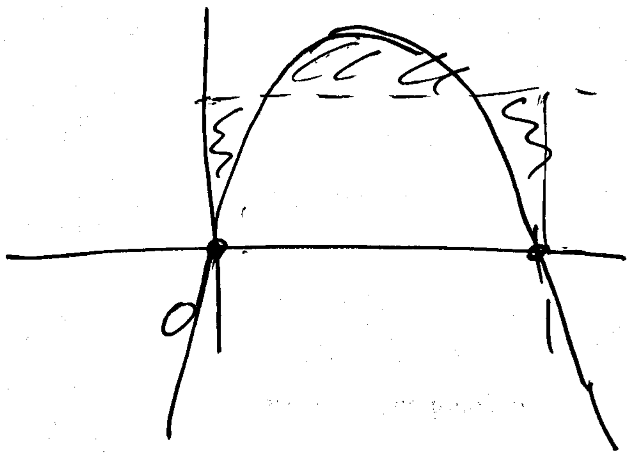
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = 2\int_0^{\frac{\pi}{2}} \cos x\,dx$$

$$= 2\sin x\Big|_0^{\frac{\pi}{2}} = 2\left(\sin\frac{\pi}{2} - 0\right)$$

$$= 2$$

$$f_{av} = \frac{1}{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)} \cdot 2 = \frac{2}{\pi} \,//$$

22) $f(x) = x(1-x)$ $\quad [0,1]$



$$f_{av} = \left(\frac{1}{1-0}\right)\int_0^1 x(1-x)\,dx$$

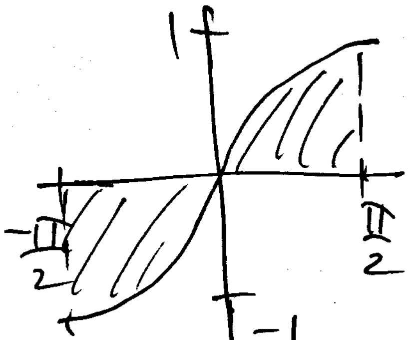
$$= \int_0^1 x(1-x)\,dx$$

$$= \int_0^1 (x - x^2)\,dx$$

$$= \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1 = \left(\frac{1}{2} - \frac{1}{3}\right) - (0)$$

$$= \frac{1}{6} \quad //$$

<u>e.g.</u> avg value of $\sin(x)$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



0

#12) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\cos 2x + \cos x \sin x - 3\sin(x^5)\right) dx$

Know: If $f(x)$ is even: $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

If $f(x)$ is odd: $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$

$\cos x \sin x$ is odd so $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \sin x\,dx = 0$

$\sin(x^5)$ is odd: $\sin((-x)^5) = \sin(-(x^5))$

$= -\sin(x^5)$

so $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3\sin(x^5)\,dx = 0.$

$$= 2\int_0^{\pi/2} \cos(2x)\,dx$$

#39) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\,dx = 2\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$

$$= 2\tan x \Big|_0^{\frac{\pi}{4}} = 2\left(\tan\tfrac{\pi}{4} - \cancel{\tan 0}^{\,0}\right) = 2$$

#41) $\displaystyle\int_{-2}^{2} \frac{x^3-4x}{x^2+1}\,dx = 0$ since ftn is odd.

5.5 Substitution Rule.

- **eg** $\int 2x(x^2+1)^{1/2}\,dx$ can't do it.

How can we attack this.

Chain rule: $\frac{d}{dx}f(g(x)) = f'(g(x))\,g'(x)$

Notice: $g(x) = x^2+1$ $\quad g'(x) = 2x$

Easier way: $u = x^2+1$ $\quad \frac{du}{dx} = 2x$

or $du = 2x\,dx$

How this works:

$$\int 2x\,\boxed{(x^2+1)^{1/2}}\,dx \qquad u = x^2+1 \qquad du = 2x\,dx$$

$2x \ldots dx \to du$

$(x^2+1)^{1/2} \to u^{1/2}$

$$\int u^{1/2}\,du = \frac{2}{3}u^{3/2} + c$$

$$= \frac{2}{3}(x^2+1)^{3/2} + c$$

I have reversed the chain rule!

$$\frac{d}{dx}\left(\frac{2}{3}(x^2+1)^{3/2}\right) = \frac{2}{3}\cdot\underbrace{\frac{3}{2}(x^2+1)^{1/2}}_{\frac{3}{2}(g(x))^{1/2}}\underbrace{(2x)}_{g'(x)} = (x^2+1)\cdot 2x$$

e.g. $\int (4x-1)^{1/2}\,dx$ $\quad u = 4x-1$

$du = 4\,dx$

$= \frac{1}{4}\int \underbrace{(4x-1)}_{u}{}^{1/2}\,\underbrace{(4\,dx)}_{du}$

$= \frac{1}{4}\int u^{1/2}\,du = \frac{1}{\cancel{4}_{2}}\cdot\frac{\cancel{2}}{3}u^{3/2} + c$

$= \frac{1}{6}(4x-1)^{3/2} + c$

e.g. $\int \cos(2x+5)\,dx$ $\quad u = 2x+5$

$du = 2\,dx$

$= \frac{1}{2}\int \cos(2x+5)\,(2\,dx)$

$= \frac{1}{2}\int \cos(u)\,du = \frac{1}{2}\sin(u) + c$

$= \frac{1}{2}\sin(2x+5) + c.$

#2d) $\int \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\,dx$ $\quad u=\sqrt{x}+1=x^{1/2}+1$

$\frac{1}{2\sqrt{x}}\,dx \to du$ $\quad du=\frac{1}{2}x^{-1/2}\,dx$

$\qquad = \frac{1}{2\sqrt{x}}\,dx$

$= \int u^4\,du = \frac{1}{5}u^5+C$

$= \frac{1}{5}(\sqrt{x}+1)^5+C.$