MATH 113 – QUIZ 8 – 6 NOVEMBER 2012

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Find all of the critical points of the function $f(x) = \dfrac{x}{x^2+1}$ on the interval $[-3, 4]$ and find the location of the absolute minimum of the function on that interval.

$$f'(x) = \frac{(x^2+1)(1) - (x)(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$$\frac{1-x^2}{(x^2+1)^2} = 0 \Longrightarrow \boxed{x=1,\ x=-1 \text{ (critical pts)}}$$

$$f(-3) = -\frac{3}{10} \quad f(-1) = -\frac{1}{2} \quad f(1) = \frac{1}{2} \quad f(4) = \frac{4}{17}$$

Absolute minimum at $x=-1$

2. (5 pts.) Consider the function $f(x) = x^{2/3}(4 - x^2)$ defined on the interval $[-3, 4]$. Find the intervals of increase and decrease for f.

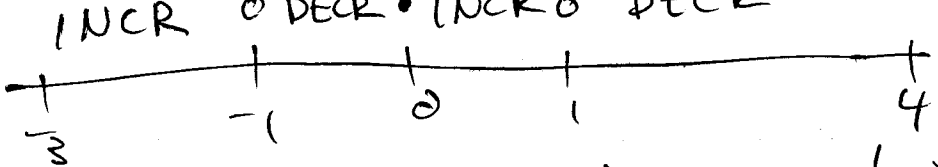
$$f'(x) = x^{2/3}(-2x) + \frac{2}{3}x^{-1/3}(4-x^2)$$

$$= -2x^{5/3} + \frac{8}{3}x^{-1/3} - \frac{2}{3}x^{5/3}$$

$$= \frac{8}{3}x^{-1/3} - \frac{8}{3}x^{5/3} = \frac{8}{3}x^{-1/3}(1-x^2)$$

$\frac{8}{3}x^{-1/3}(1-x^2)$ undefined at $x=0$

$\frac{8}{3}x^{-1/3}(1-x^2) = 0 \Longrightarrow x=1,\ x=-1$



$f'(-2) > 0 \quad f'(-\frac{1}{2}) < 0 \quad f'(\frac{1}{2}) > 0 \quad f'(4) < 0$

INCR: $[-3,-1) \cup (0,1)$

DECR: $(-1,0) \cup [1,4]$

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1. (5 pts.) Find all of the critical points of the function $f(x) = x^{2/3}(4 - x^2)$ on the interval $[-3, 4]$ and find the location of the absolute maximum of the function on that interval.

$$f'(x) = x^{2/3}(-2x) + \frac{2}{3}x^{-1/3}(4-x^2)$$

$$= -2x^{5/3} + \frac{8}{3}x^{-1/3} - \frac{2}{3}x^{5/3}$$

$$= \frac{8}{3}x^{-1/3} - \frac{8}{3}x^{5/3} = \frac{8}{3}x^{-1/3}(1-x^2)$$

Critical points: $x=0, x=-1, x=1$

$f(-3) = 9^{1/3}(4-9) = -5\sqrt[3]{9}$

$f(0) = 0$ $\quad f(-1) = 3$

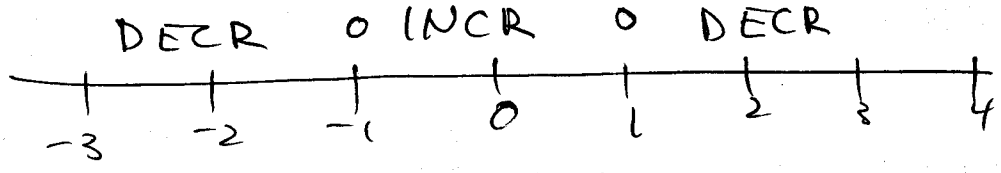
$f(1) = 3$ $\quad f(4) = -12\sqrt[3]{16}$

Absolute max at $x=1, x=-1$

2. (5 pts.) Consider the function $f(x) = \dfrac{x}{x^2+1}$ defined on the interval $[-3, 4]$. Find the intervals of increase and decrease for f.

$$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$f'(x) = 0$ if $x = -1, 1$



$f'(0) > 0$ $\quad f'(2) < 0$ $\quad f'(-2) < 0$

INCR on $(-1, 1)$

DECR on $[-3, -1) \cup (1, 4]$ //

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Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Find all of the critical points of the function $f(x) = x(x-2)^{1/3}$ on the interval $[-3, 4]$ and find the location of the absolute maximum of the function on that interval.

$f'(x) = x \cdot \frac{1}{3}(x-2)^{-2/3} + (x-2)^{1/3} = \frac{x + 3x - 6}{3(x-2)^{2/3}}$

$= \frac{4x-6}{3(x-2)^{2/3}}$

CP: $x = \frac{3}{2}, 2$

$f(-3) = 3\sqrt[3]{5}$ $f(\frac{3}{2}) = \frac{-3}{2\sqrt[3]{2}}$ $f(2) = 0$ $f(4) = 4\sqrt[3]{2}$

Abs max at $x = -3$

2. (5 pts.) Consider the function $f(x) = \frac{x}{x^2+1}$ defined on the interval $[-3, 4]$. Find the intervals of increase and decrease for f.

Same as other quiz