VER 1

MATH 113 – QUIZ 4 – 25 SEPTEMBER 2012

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Assume that $\lim_{x\to 3}(2x-1) = 5$ (see the graph). For the value of $\epsilon = 1$, find a value of $\delta > 0$ such that $\cancel{|(2x+3)-7| < \epsilon}$ whenever $\cancel{0 < |x-2| < \delta}$.

$|(2x-1)-5| < \varepsilon \qquad 0 < |x-3| < \delta$

Taking $y = 2x-1$, $|(2x-1)-5| < \varepsilon$ is the same as $|y-5| < \varepsilon$ which is the same as $y \in (4,6)$ if $\varepsilon = 1$. From the graph, we see that if $x \in (2.5, 3.5)$ then $y \in (4,6)$. But $x \in (2.5, 3.5)$ is the same as $|x-3| < \frac{1}{2}$

$\boxed{\text{so } \delta = \tfrac{1}{2} \text{ works}}$

2. (5 pts.) Let $f(x) = 3x^2 - 5x$, and let $a = 3$.

(a) (3 pts.) Find $f'(a)$ by computing the limit of the difference quotient. Do not use any shortcuts you may be aware of.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{3(3+h)^2-5(3+h)-12}{h}$$

$$= \lim_{h\to 0} \frac{3(9+6h+h^2)-(15+5h)-12}{h} = \lim_{h\to 0} \frac{\cancel{27}+18h+3h^2-\cancel{15}-5h-\cancel{12}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(18+3h-5)}{\cancel{h}} = 13 //$$

(b) (2 pts.) Find the equation of the tangent line to the graph of $f(x)$ at a.

$(a, f(a)) = (3, f(3)) = (3, 12)$

Equ of tangent line: $y - 12 = 13(x-3)$

$y = 13x - 27 //$

VER 2

MATH 113 – QUIZ 4 – 25 SEPTEMBER 2012

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Assume that $\lim_{x\to 3}(2x-1) = 5$ (see the graph). For the value of $\epsilon = 2$, find a value of $\delta > 0$ such that ~~$|(2x-1)-5|<\epsilon$ whenever $0<|x-3|<\delta$~~ $|(2x-1)-5|<\varepsilon$ whenever $0<|x-3|<\delta$.

Letting $y=2x-1$, $|y-5|<\varepsilon$ means $y\in(5-\varepsilon, 5+\varepsilon)$ and since $\varepsilon=2$ this means $y\in(3,7)$. By the graph, if $x\in(2,4)$ then $y\in(3,7)$. Since $x\in(2,4)$ means $|x-3|<1$ Then $\boxed{\delta=1 \text{ works}}$

2. (5 pts.) Let $f(x) = \sqrt{x+1}$, and let $a = 3$.

(a) (3 pts.) Find $f'(a)$ by computing the limit of the difference quotient. Do not use any shortcuts you may be aware of.

$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\sqrt{3+h+1}-2}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \lim_{h\to 0} \frac{(\cancel{4}+h)-\cancel{4}}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)} = \frac{1}{\sqrt{4}+2} = \frac{1}{4} //$$

(b) (2 pts.) Find the equation of the tangent line to the graph of $f(x)$ at a.

$(a, f(a)) = (3, f(3)) = (3, 2)$

Equ of tangent line: $y-2 = \frac{1}{4}(x-3)$

$y = \frac{1}{4}x + \frac{5}{4}$ //

VER 3

MATH 113 – QUIZ 4 – 25 SEPTEMBER 2012

Answer all of the following questions in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit.

1. (5 pts.) Assume that $\lim_{x\to 3}(2x-1) = 5$ (see the graph). For the value of $\epsilon = 3$, find a value of $\delta > 0$ such that ~~[illegible]~~. $|(2x-1)-5| < \varepsilon$ whenever $0 < |x-3| < \delta$.

Letting $y = 2x-1$, $|y-5| < \varepsilon$ means $y \in (5-\varepsilon, 5+\varepsilon)$ or when $\varepsilon = 3$, $y \in (2,8)$. By the graph, if $x \in (1.5, 4.5)$ then $y \in (2,8)$. But $x \in (1.5, 4.5)$ means $|x-3| < 1.5$

Therefore $\boxed{\delta = 1.5 \text{ works}}$

2. (5 pts.) Let $f(x) = \dfrac{2}{x}$, and let $a = 2$.

(a) (3 pts.) Find $f'(a)$ by computing the limit of the difference quotient. Do not use any shortcuts you may be aware of.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{\frac{2}{2+h} - 1}{h}$$

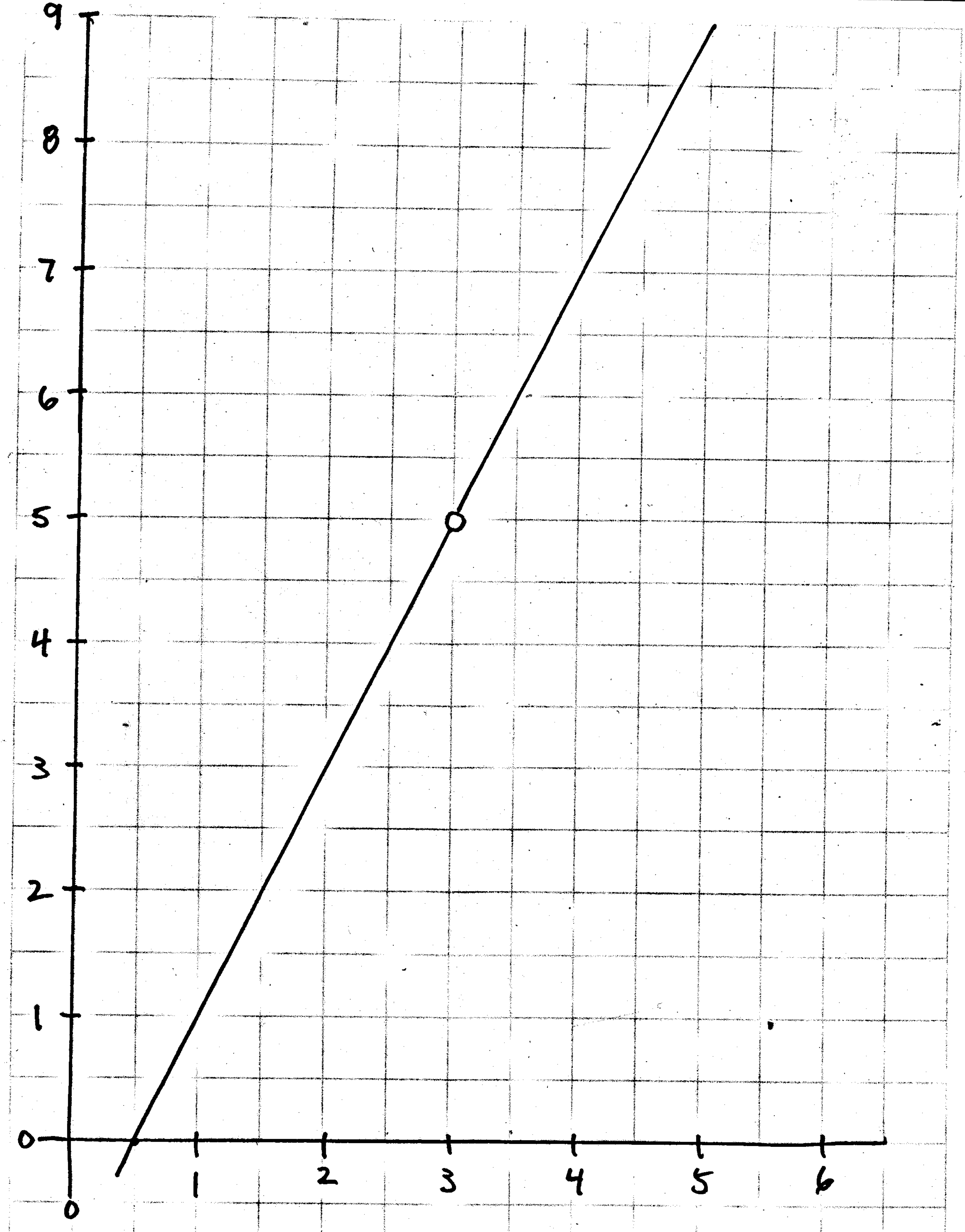
$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{2-(2+h)}{2+h}\right) = \lim_{h\to 0} \frac{1}{\cancel{h}}\left(\frac{-\cancel{h}}{2+h}\right) = -\frac{1}{2}$$

(b) (2 pts.) Find the equation of the tangent line to the graph of $f(x)$ at a.

$(a, f(a)) = (2, f(2)) = (2, 1)$

Eqn of tangent line: $y - 1 = -\frac{1}{2}(x-2)$

$$y = -\frac{1}{2}x + 2$$



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