

MATH 113 – 28 NOVEMBER 2012 – EXAM 3

Answer each of the following questions. Show all work, as partial credit may be given. This exam is counted out of a total of 90 points.

1. (10 pts. each) Let $f(x) = x^4 + 8x^3 - 270x^2 + 1$.
 - (a) Find the intervals on which $f(x)$ is increasing and decreasing, and the location of all local maxima and minima for $f(x)$.
 - (b) Find the intervals on which $f(x)$ is concave up and concave down, and the location of all inflection points of $f(x)$.
2. (10 pts.) A box with a square base must have a volume of 100 m^3 . Find the dimensions of the box that minimize its surface area. What is the minimum surface area?
3. (10 pts.) Find the linear approximation of the function $f(x) = \sin(x)$ at $a = \pi/6$ and use it to estimate $\sin(2\pi/9)$. (You may leave your answer in terms of π).
4. (10 pts. each) Find the following limits using L'Hôpital's rule.

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{(8 - x^2)^{1/2} - x}.$

(b) $\lim_{x \rightarrow \infty} \left(1 + \frac{4}{x}\right)^{3x}.$

5. (10 pts. each) Compute the following antiderivatives.

(a) $\int (2x^4 - x^3 + 5x^{1/2} - \frac{6}{x}) dx.$

(b) $\int (\cos(2y) + \sin(3y)) dy.$

6. (10 pts.) Solve the following initial value problem. $f'(x) = 4x^{1/3} + 6x^{-1/2}$, $f(1) = 2$.