Answer each of the following questions. Show all work, as partial credit may be given. This exam is counted out of a total of 90 points.

1. (10 pts. each) Let $f(x) = 5x^4 - x^5$.

   (a) Find the intervals on which $f(x)$ is increasing and decreasing, and the location of all local maxima and minima for $f(x)$.

   (b) Find the intervals on which $f(x)$ is concave up and concave down, and the location of all inflection points of $f(x)$.

2. (10 pts.) A rectangular pen is built with one side against a barn. 200 meters of fencing are used for the other three sides of the pen. What dimensions maximize the area of the pen? What is this maximum area?

3. (10 pts.) Find the linear approximation of the function $f(x) = x^{1/3}$ at $a = 64$ and use it to estimate $(65)^{1/3}$.

4. (10 pts. each) Find the following limits using L'Hopital's rule.

   (a) $\displaystyle\lim_{x \to 2} \frac{(3x+2)^{1/3} - 2}{x - 2}$.

   (b) $\displaystyle\lim_{x \to 0} (1 + 4x)^{3/x}$.

5. (10 pts. each) Compute the following antiderivatives.

   (a) $\displaystyle\int \left(2x^4 - x^3 + 5x^{1/2} - \frac{6}{x}\right) dx$.

   (b) $\displaystyle\int \left(\cos(2y) + \sin(3y)\right) dy$.

6. (10 pts.) Solve the following initial value problem. $f'(x) = 4x^{1/3} + 6x^{-1/2}$, $f(1) = 2$.