MATH 113 — EXAM 3 — SOLUTIONS

1. (a) $f'(x) = 4x^3 + 24x^2 - 540x$

$4x^3 + 24x^2 - 540x = 0$

$4x(x^2 + 6x - 135) = 0$

$4x(x-9)(x+15) = 0$

$x = 0 \quad x = 9 \quad x = -15$ critical points

DECR o INCR o DECR o INCR. f'

$-15 \qquad 0 \qquad 9$

$f'(-16) < 0 \quad f'(-1) > 0 \quad f'(1) < 0 \quad f'(10) > 0$

f increasing on $(-15, 0) \cup (9, \infty)$

f decreasing on $(-\infty, -15) \cup (0, 9)$.

local minimum at $x = -15$, $x = 9$

local maximum at $x = 0$

(b) $f''(x) = 12x^2 + 48x - 540$

$12x^2 + 48x - 540 = 0$

$12(x^2 + 4x - 45) = 0$

$12(x+9)(x-5) = 0$

possible inflection points at $x = 5$, $x = -9$

CONC UP ∘ CONC DN ∘ CONC UP

$\underset{-9}{|} \qquad \underset{0}{|} \qquad \underset{5}{|} \qquad f''$

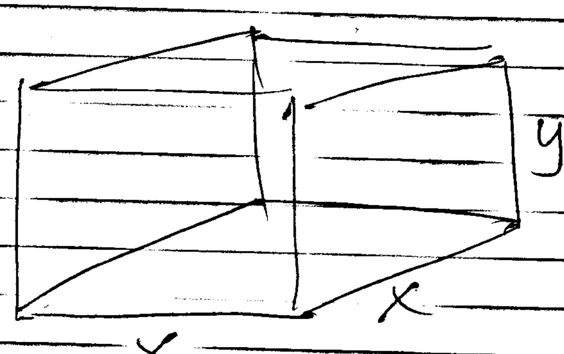
$f''(-10) > 0 \quad f''(0) < 0 \quad f''(6) > 0$

f concave up on $(-\infty, -9) \cup (5, \infty)$

f concave down on $(-9, 5)$

inflection points at $x = -9$, $x = 5$ //

2.



$100 = x^2 y$

$S = 2x^2 + 4xy$

$xy = \frac{100}{x}$

$S = 2x^2 + \frac{400}{x}$

$S' = 4x - \frac{400}{x^2}$

$4x - \frac{400}{x^2} = 0$

$4x^3 = 400$

$x^3 = 100$

$x = (100)^{1/3}$

$y = \frac{100}{(100)^{2/3}} = (100)^{1/3}$ //

$S = 2(100)^{2/3} + 4(100)^{2/3}$

$= 6(100)^{2/3}$ //.

3. $f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$f'(x) = \cos(x)$

$f'\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$L(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

$\sin\left(\frac{2\pi}{9}\right) = f\left(\frac{2\pi}{9}\right) \approx L\left(\frac{2\pi}{9}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{2\pi}{9} - \frac{\pi}{6}\right)$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\pi}{18} = \frac{1}{2} + \frac{\pi\sqrt{3}}{36}$ //

4. (a) $\lim\limits_{x \to 2} \dfrac{x^2 - 2x}{(8-x^2)^{1/2} - x} = \lim\limits_{x \to 2} \dfrac{2x - 2}{\frac{1}{2}(8-x^2)^{-1/2}(-2x) - 1}$

$= \lim\limits_{x \to 2} \dfrac{2(x-1)}{-\frac{x}{(8-x^2)^{1/2}} - 1} = \dfrac{2}{-2} = -1$

(b) $\lim\limits_{x \to \infty} \left(1 + \frac{4}{x}\right)^{3x} = e^{12}$ //

$\ln\left(\left(1 + \frac{4}{x}\right)^{3x}\right) = 3x \ln\left(1 + \frac{4}{x}\right) = \dfrac{3\ln\left(1 + \frac{4}{x}\right)}{\frac{1}{x}}$

$\lim\limits_{x \to \infty} \dfrac{3\ln\left(1 + \frac{4}{x}\right)}{\frac{1}{x}} = \lim\limits_{x \to \infty} \dfrac{3 \cdot \frac{1}{1 + \frac{4}{x}} \cdot \frac{-4}{x^2}}{\frac{-1}{x^2}}$

$= \lim\limits_{x \to \infty} \dfrac{12}{1 + \frac{4}{x}} = 12$ (see above)

5. (a) $\int \left(2x^4 - x^3 + 5x^{1/2} - \frac{6}{x}\right) dx$

$= \frac{2}{5}x^5 - \frac{1}{4}x^4 + 5 \cdot \frac{2}{3}x^{3/2} - 6\ln|x| + C$

$= \frac{2}{5}x^5 - \frac{1}{4}x^4 + \frac{10}{3}x^{3/2} - 6\ln|x| + C$ //

(b) $\int \left(\cos(2y) + \sin(3y)\right) dy$

$= \frac{1}{2}\sin(2y) - \frac{1}{3}\cos(3y) + C$ //.

6. $f(x) = \int \left(4x^{1/3} + 6x^{-1/2}\right) dx$

$= 3x^{4/3} + 12x^{1/2} + C.$

$2 = f(1) = 3 + 12 + C = 15 + C$

$\therefore C = -13$ // $\quad f(x) = 3x^{4/3} + 12x^{1/2} - 13$ //

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1. (a) $f'(x) = 20x^3 - 5x^4$

$20x^3 - 5x^4 = 0$

$5x^3(4 - x) = 0$

$x = 0 \quad x = 4$ critical points

DECR 0 INCR 0 DECR f'

0 4

$f'(-1) < 0 \quad f'(1) > 0 \quad f'(5) < 0$

f increasing on $(0, 4)$

f decreasing on $(-\infty, 0) \cup (4, \infty)$

local min at $x = 0$

local max at $x = 4$

(b) $f''(x) = 60x^2 - 20x^3$

$60x^2 - 20x^3 = 0$

$20x^2(3 - x) = 0$

$x = 0 \quad x = 3$ possible inflection points

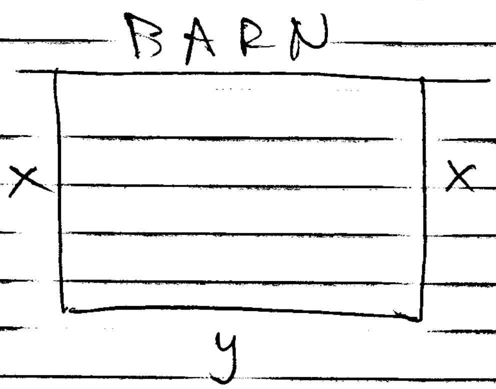
CONC UP 0 CONC UP 0 CONC DN. $f''(x)$

0 3

$f''(-1) > 0 \quad f''(1) > 0 \quad f''(4) < 0$

f is concave up on $(-\infty, 3)$
f is concave down on $(3, \infty)$
inflection point at $x=3$ //

2.



$A = xy$

$200 = 2x + y$

$y = 200 - 2x$

$A = x(200-2x) = -2x^2 + 200x$

$A' = -4x + 200 \qquad -4x + 200 = 0$

$\underline{x = 50} \qquad \underline{y = 100}$ //

$\underline{A = 5000}$

3. $f(64) = 64^{1/3} = 4$

$f'(x) = \frac{1}{3}x^{-2/3} \qquad f'(64) = \frac{1}{3}(64)^{-2/3} = \frac{1}{3}\cdot\frac{1}{16}$
$= 1/48$ //

$L(x) = 4 + \frac{1}{48}(x - 64)$

$65^{1/3} \approx L(65) = 4 + \frac{1}{48} = \frac{193}{48}$ //

4. (a) $\lim\limits_{x\to 2} \dfrac{(3x+2)^{1/3}-2}{x-2} = \lim\limits_{x\to 2} \dfrac{\frac{1}{3}(3x+2)^{-2/3}\cdot 3}{1}$

$= \lim\limits_{x\to 2} (3x+2)^{-2/3} = 8^{-2/3} = \dfrac{1}{4}$ //

(b) $\lim\limits_{x\to 0^+} (1+4x)^{3/x}$

$\ln\left[(1+4x)^{3/x}\right] = \dfrac{3}{x} \ln(1+4x) = \dfrac{3\ln(1+4x)}{x}$

$\lim\limits_{x\to 0^+} \dfrac{3\ln(1+4x)}{x} = \lim\limits_{x\to 0^+} \dfrac{\frac{3\cdot 4}{1+4x}}{1} = \lim\limits_{x\to 0} \dfrac{12}{1+4x}$

$= 12.$

$\therefore \lim\limits_{x\to 0^+} (1+4x)^{3/x} = e^{12}$

5. (a) $\int \left(2x^4 - x^3 + 5x^{1/2} - \frac{6}{x}\right) dx$

$= \frac{2}{5}x^5 - \frac{1}{4}x^4 + 5 \cdot \frac{2}{3}x^{3/2} - 6 \ln|x| + C$

$= \frac{2}{5}x^5 - \frac{1}{4}x^4 + \frac{10}{3}x^{3/2} - 6 \ln|x| + C$ //

(b) $\int \left(\cos(2y) + \sin(3y)\right) dy$

$= \frac{1}{2}\sin(2y) - \frac{1}{3}\cos(3y) + C$ //.

6. $f(x) = \int \left(4x^{1/3} + 6x^{-1/2}\right) dx$

$= 3x^{4/3} + 12x^{1/2} + C.$

$2 = f(1) = 3 + 12 + C = 15 + C$

$\therefore C = -13$ // $f(x) = 3x^{4/3} + 12x^{1/2} - 13$ //