

# MATH 113 - EXAM 2 VER 1 - SOLUTIONS

$$1. f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2(4+h)+1} - 3}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{9+2h} - 3}{h} \cdot \frac{\sqrt{9+2h} + 3}{\sqrt{9+2h} + 3}$$

$$= \lim_{h \rightarrow 0} \frac{(9+2h) - 9}{h(\sqrt{9+2h} + 3)} = \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{9+2h} + 3)}$$

$$= \lim_{h \rightarrow 0} \frac{2}{\sqrt{9+2h} + 3} = \frac{2}{6} = \frac{1}{3} //$$

Eqn of line:  $y - 3 = \frac{1}{3}(x - 4)$

$$y = \frac{1}{3}x + \frac{5}{3} //$$

2. (a)  $f'(x) = 8x^3 - 3x^2 + 10x$

(b)  $g'(t) = \frac{(t^2-1)(2t+1) - (t^2+t+2)(2t)}{(t^2-1)^2}$

$$= \frac{2t^3 - 2t + t^3 - 1 - 2t^3 - 2t^2 - 4t}{(t^2-1)^2}$$

$$= \frac{-t^2 - 6t - 1}{(t^2-1)^2} //$$

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$$(c) \quad r'(y) = y^2 (\cos(y)) + \sin(y) \cdot 2y$$

$$= y^2 \cos(y) + 2y \sin(y)$$

$$(d) \quad f'(t) = 2 \cos(2^t) \frac{d}{dt} \cos(2^t)$$

$$= 2 \cos(2^t) (-\sin(2^t) \frac{d}{dt} (2^t))$$

$$= 2 \cos(2^t) (-\sin(2^t)) (2^t \ln 2)$$

$$= -2 \ln(2) 2^t \cos(2^t) \sin(2^t) //$$

$$(e) \quad s'(x) = 5 (\ln(x^2) + 1)^4 \left( \frac{2x}{x^2} \right)$$

$$= \frac{10}{x} (\ln(x^2) + 1)^4 //$$

$$3. (a) \quad v(t) = s'(t) = 6t^2 - 42t + 60$$

$$s''(t) = 12t - 42$$

$$6t^2 - 42t + 60 = 0$$

$$6(t^2 - 7t + 10) = 0$$

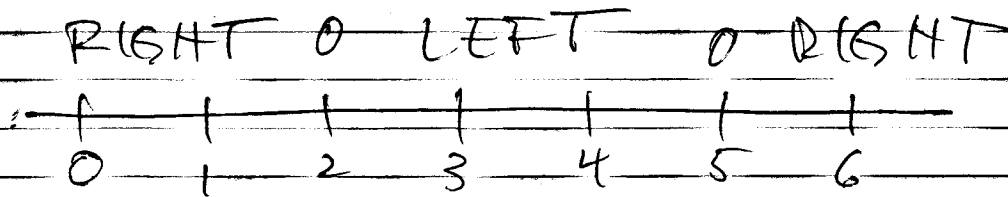
$$6(t-2)(t-5) = 0$$

$$t=2 \quad t=5 \quad \leftarrow \text{velocity} = 0 \quad -2 \text{ of } 4-$$

$$s''(2) = 24 - 42 = -18$$

$$s''(5) = 60 - 42 = 18$$

(b)



$$s'(1) = 6(-1)(-4) > 0$$

$$s'(3) = 6(1)(-2) < 0$$

$$s'(6) = 6(4)(1) > 0$$

Moving right on

$$[0, 2) \cup (5, 6]$$

Moving left on

$$[2, 5]$$

$$4. f'(x) = x \cdot \frac{1}{1+x^2} + (1) \tan^{-1}(x)$$

$$= \frac{x}{1+x^2} + \tan^{-1}(x) //$$

$$f''(x) = \frac{(1+x^2)(1) - (x)(2x)}{(1+x^2)^2} + \frac{1}{1+x^2}$$

$$= \frac{1-x^2}{(1+x^2)^2} + \frac{1+x^2}{(1+x^2)^2} = \frac{2}{(1+x^2)^2} //$$

$$5. \quad xy^{1/3} + y = 10$$

$$\frac{d}{dx}(xy^{1/3} + y) = \frac{d}{dx}(10)$$

$$x \frac{d}{dx}(y^{1/3}) + y^{1/3} \frac{d}{dx}(x) + \frac{d}{dx}(y) = 0$$

$$x \cdot \frac{1}{3} y^{-2/3} \frac{dy}{dx} + y^{1/3} + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \left( 1 + \frac{x}{3} y^{-2/3} \right) = -y^{1/3}$$

$$\frac{dy}{dx} = \frac{-y^{1/3}}{1 + \frac{x}{3} y^{-2/3}} \cdot \frac{3y^{2/3}}{3y^{2/3}} = \frac{-3y}{3y^{2/3} + x} //$$

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# MATH 1B - EXAM 2, VER 2 - SOLUTIONS

$$1. f'(-1) = \lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h \rightarrow 0} \frac{\frac{2}{3(-1+h)+1} - (-1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{2}{3h-2} + 1}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{2+3h-2}{3h-2} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{3h}{3h-2} \right) = -\frac{3}{2}$$

Equ of line:  $y - (-1) = -\frac{3}{2}(x - (-1))$

$$y + 1 = -\frac{3}{2}(x + 1)$$

$$y = -\frac{3}{2}x - \frac{5}{2} //$$

2. (a)  $f'(x) = 4x^3 - 6x^2 + 12x$

(b)  $g'(t) = (t^2 + t + 2)(2t) + (2t + 1)(t^2 - 1)$

$$= 2t^3 + 2t^2 + 4t + 2t^3 + t^2 - 2t - 1$$

$$= 4t^3 + 3t^2 + 2t - 1 //$$

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$$(c) \quad r'(y) = \frac{y(3 \cos(3y)) - \sin(3y)}{y^2}$$

$$= \frac{3y \cos(3y) - \sin(3y)}{y^2} //$$

$$(d) \quad f'(t) = 2 \cos(\log_2 t) \frac{d}{dt}(\cos(\log_2 t))$$

$$= 2 \cos(\log_2 t) (-\sin(\log_2 t)) \frac{d}{dt}(\log_2 t)$$

$$= 2 \cos(\log_2 t) (-\sin(\log_2 t)) \frac{1}{t \ln 2}$$

$$= \frac{-2}{t \ln 2} \cos(\log_2 t) \sin(\log_2 t) //$$

$$(e) \quad s'(x) = \frac{1}{2} (e^{x^2} + 1)^{-1/2} \frac{d}{dx}(e^{x^2})$$

$$= \frac{1}{2} (e^{x^2} + 1)^{-1/2} (2x e^{x^2})$$

$$= x e^{x^2} (e^{x^2} + 1)^{-1/2} //$$

$$3 \text{ (a)} \quad v(t) = s'(t) = -6t^2 + 24t - 18$$

$$s'(2) = -24 + 48 - 18 = 6 //$$

$$a(t) = s''(t) = -12t + 24$$

$$s''(2) = 0 //$$

The object is neither speeding up nor slowing down.

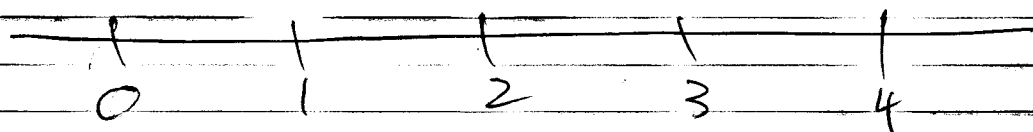
$$(b) \quad -6t^2 + 24t - 18 = 0$$

$$-6(t^2 - 4t + 3) = 0$$

$$-6(t-1)(t-3) = 0$$

$$t=1, t=3 \quad \leftarrow \text{velocity is zero}$$

LEFT 0 RIGHT 0 LEFT



$$s'(0) = -18 < 0 \quad s'(2) = 6 > 0 \quad s'(4) = -18 < 0$$

Moving left on  $[0, 1) \cup (3, 4]$

Moving right on  $(1, 3)$  //

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$$4. \quad f'(x) = \frac{1}{\sqrt{1-x^2}} \cdot 2x = \frac{2x}{(1-x^2)^{1/2}}$$

$$f''(x) = \frac{(1-x^2)^{1/2} (2) - (2x) (\frac{1}{2}(1-x^2)^{-1/2} (-4x^3))}{(1-x^2)^2}$$

$$= \frac{2(1-x^2)^{1/2} + 4x^4(1-x^2)^{-1/2}}{(1-x^2)^2} = \frac{(1-x^2)^{1/2}}{(1-x^2)^{3/2}}$$

$$= \frac{2(1-x^2) + 4x^4}{(1-x^2)^{3/2}} = \frac{2(1+x^2)}{(1-x^2)^{3/2}}$$

$$5. \quad (x+y)^{2/3} = y$$

$$\frac{d}{dx} (x+y)^{2/3} = \frac{d}{dx} y$$

$$\frac{dy}{dx} = \frac{-\frac{2}{3}(x+y)^{-1/3} (\frac{2}{3}(x+y)^{1/3})}{(\frac{2}{3}(x+y)^{-1/3} - 1)(-\frac{2}{3}(x+y)^{1/3})}$$

$$\frac{2}{3}(x+y)^{-1/3} \frac{d}{dx} (x+y) = \frac{dy}{dx} = \frac{2}{-2 + 3(x+y)^{1/3}}$$

$$\frac{2}{3}(x+y)^{-1/3} (1 + \frac{dy}{dx}) = \frac{dy}{dx}$$

$$\frac{2}{3}(x+y)^{-1/3} + \frac{2}{3}(x+y)^{-1/3} \frac{dy}{dx} = \frac{dy}{dx}$$

$$\frac{dy}{dx} (\frac{2}{3}(x+y)^{-1/3} - 1) = -\frac{2}{3}(x+y)^{-1/3}$$

- 4 or 4 -