MATH 113 — EXAM 2 VER 1 — SOLUTIONS

1. $$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{f(4+h)-f(4)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(4+h)+1}-3}{h} = \lim_{h\to 0} \frac{\sqrt{9+2h}-3}{h} \cdot \frac{\sqrt{9+2h}+3}{\sqrt{9+2h}+3}$$

$$= \lim_{h\to 0} \frac{(9+2h)-9}{h(\sqrt{9+2h}+3)} = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{9+2h}+3)}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{9+2h}+3} = \frac{2}{6} = \frac{1}{3} //$$

Eqn of line: $y - 3 = \frac{1}{3}(x-4)$

$$y = \frac{1}{3}x + \frac{5}{3} //$$

2. (a) $f'(x) = 8x^3 - 3x^2 + 10x$

(b) $$g'(t) = \frac{(t^2-1)(2t+1)-(t^2+t+2)(2t)}{(t^2-1)^2}$$

$$= \frac{\cancel{2t^3}-2t+t^2-1-\cancel{2t^3}-2t^2-4t}{(t^2-1)^2}$$

$$= \frac{-t^2-6t-1}{(t^2-1)^2} //$$

(c) $r'(y) = \underline{y^2(\cos(y))} + \sin(y) \cdot 2y$

$= y^2\cos(y) + 2y\sin(y)$

(d) $f'(t) = 2\cos(2^t)\frac{d}{dt}\cos(2^t)$

$= 2\cos(2^t)\left(-\sin(2^t)\frac{d}{dt}(2^t)\right)$

$= 2\cos(2^t)(-\sin(2^t))(2^t \ln 2)$

$= -2\ln(2)\, 2^t \cos(2^t)\sin(2^t)$ //

(e) $s'(x) = 5(\ln(x^2)+1)^4\left(\frac{2x}{x^2}\right)$

$= \frac{10}{x}(\ln(x^2)+1)^4$ //

3. (a) $v(t) = s'(t) = 6t^2 - 42t + 60$

$s''(t) = 12t - 42$

$6t^2 - 42t + 60 = 0$

$6(t^2 - 7t + 10) = 0$

$6(t-2)(t-5) = 0$

$$\boxed{\begin{array}{l} s''(2) = 24 - 42 = -18 \\ s''(5) = 60 - 42 = 18 \end{array}}$$

$t = 2 \quad t = 5 \leftarrow$ velocity $= 0$

(b)

RIGHT 0 LEFT 0 RIGHT

0 1 2 3 4 5 6

$s'(1) = 6(-1)(-4) > 0$

$s'(3) = 6(1)(-2) < 0$

$s'(6) = 6(4)(1) > 0$

$\boxed{\begin{array}{l}\text{Moving right on} \\ [0,2) \cup (5,6] \\ \text{Moving left on} \\ [2,5]\end{array}}$

4. $f'(x) = x \cdot \dfrac{1}{1+x^2} + (1)\tan^{-1}(x)$

$$= \frac{x}{1+x^2} + \tan^{-1}(x) \quad //$$

$$f''(x) = \frac{(1+x^2)(1) - (x)(2x)}{(1+x^2)^2} + \frac{1}{1+x^2}$$

$$= \frac{1-x^2}{(1+x^2)^2} + \frac{1+x^2}{(1+x^2)^2} = \frac{2}{(1+x^2)^2} \quad //$$

5. $xy^{1/3} + y = 10$

$\frac{d}{dx}(xy^{1/3} + y) = \frac{d}{dx}(10)$

$x\frac{d}{dx}(y^{1/3}) + y^{1/3}\frac{d}{dx}(x) + \frac{d}{dx}(y) = 0$

$x \cdot \frac{1}{3}y^{-2/3}\frac{dy}{dx} + y^{1/3} + \frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(1 + \frac{x}{3}y^{-2/3}\right) = -y^{1/3}$

$\frac{dy}{dx} = \frac{-y^{1/3}}{1 + \frac{x}{3}y^{-2/3}} \cdot \frac{3y^{2/3}}{3y^{2/3}} = \frac{-3y}{3y^{2/3} + x}$ //

MATH 113 - EXAM 2, VER 2 - SOLUTIONS

1. $f'(-1) = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h \to 0} \frac{\frac{2}{3(-1+h)+1} - (-1)}{h}$

$= \lim_{h \to 0} \frac{\frac{2}{3h-2} + 1}{h} = \lim_{h \to 0} \frac{1}{h}\left(\frac{2+3h-2}{3h-2}\right)$

$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left(\frac{3\cancel{h}}{3h-2}\right) = -\frac{3}{2}$

Equ of line: $y - (-1) = -\frac{3}{2}(x - (-1))$

$y + 1 = -\frac{3}{2}(x+1)$

$y = -\frac{3}{2}x - \frac{5}{2}$ //

2. (a) $f'(x) = 4x^3 - 6x^2 + 12x$

(b) $g'(t) = (t^2+t+2)(2t) + (2t+1)(t^2-1)$

$= 2t^3 + 2t^2 + 4t + 2t^3 + t^2 - 2t - 1$

$= 4t^3 + 3t^2 + 2t - 1$ //

(c) $r'(y) = \dfrac{y(3\cos(3y)) - \sin(3y)}{y^2}$

$= \dfrac{3y\cos(3y) - \sin(3y)}{y^2}$ //

(d) $f'(t) = 2\cos(\log_2 t)\,\frac{d}{dt}(\cos(\log_2 t))$

$= 2\cos(\log_2 t)(-\sin(\log_2 t))\,\frac{d}{dt}(\log_2 t)$

$= 2\cos(\log_2 t)(-\sin(\log_2 t))\,\dfrac{1}{t\ln 2}$

$= \dfrac{-2}{t\ln 2}\cos(\log_2 t)\sin(\log_2 t)$ //

(e) $s'(x) = \frac{1}{2}(e^{x^2}+1)^{-1/2}\,\frac{d}{dx}(e^{x^2})$

$= \frac{1}{2}(e^{x^2}+1)^{-1/2}(2xe^{x^2})$

$= xe^{x^2}(e^{x^2}+1)^{-1/2}$ //

3 (a) $v(t) = s'(t) = -6t^2 + 24t - 18$

$s'(2) = -24 + 48 - 18 = 6$ //

$a(t) = s''(t) = -12t + 24$

$s''(2) = 0$ //

The object is neither speeding up nor slowing down.

(b) $-6t^2 + 24t - 18 = 0$

$-6(t^2 - 4t + 3) = 0$

$-6(t-1)(t-3) = 0$

$t = 1, t = 3$ ← velocity is zero

LEFT 0 RIGHT 0 LEFT

0 1 2 3 4

$s'(0) = -18 < 0$ $s'(2) = 6 > 0$ $s'(4) = -18 < 0$

Moving left on $[0,1) \cup (3,4]$

Moving right on $(1,3)$ //

V2

4. $f'(x) = \dfrac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \dfrac{2x}{(1-x^4)^{1/2}}$ //

$$f''(x) = \frac{(1-x^4)^{1/2}(2) - (2x)\left(\frac{1}{2}(1-x^4)^{-1/2}(-4x^3)\right)}{(1-x^4)}$$

$$= \frac{2(1-x^4)^{1/2} + 4x^4(1-x^4)^{-1/2}}{(1-x^4)} \cdot \frac{(1-x^4)^{1/2}}{(1-x^4)^{1/2}}$$

$$= \frac{2(1-x^4) + 4x^4}{(1-x^4)^{3/2}} = \frac{2(1+x^4)}{(1-x^4)^{3/2}} \quad //$$

5. $(x+y)^{2/3} = y$

$$\frac{d}{dx}(x+y)^{2/3} = \frac{d}{dx}y$$

$$\frac{2}{3}(x+y)^{-1/3}\frac{d}{dx}(x+y) = \frac{dy}{dx}$$

$$\frac{2}{3}(x+y)^{-1/3}\left(1+\frac{dy}{dx}\right) = \frac{dy}{dx}$$

$$\frac{2}{3}(x+y)^{-1/3} + \frac{2}{3}(x+y)^{-1/3}\frac{dy}{dx} = \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{2}{3}(x+y)^{-1/3} - 1\right) = -\frac{2}{3}(x+y)^{-1/3}$$

$$\frac{dy}{dx} = \frac{-\frac{2}{3}(x+y)^{-1/3}\left(-\frac{3}{2}(x+y)^{1/3}\right)}{\left(\frac{2}{3}(x+y)^{-1/3} - 1\right)\left(-\frac{3}{2}(x+y)^{1/3}\right)}$$

$$= \frac{2}{-2+3(x+y)^{1/3}} \quad //$$